

Fractions, decimals and percentages

Progression document



EYFS Numbers – fractions

Understanding fractions

Objective: understand a half

Concrete Experience: Ask children to cut a piece of paper, or other item, into halves. Share objects equally between two. Build a tower half the size of another. Create a pattern where half are one colour/shape and half are another colour/shape. Turn half turns. Emphasise that halves should be exactly the same size.

Images:



Language: one half, halves, equal, two, parts

Symbols (when ready): $1/2$, $1 \div 2$

Questions: What is a half? How many halves make one whole? Is this a half? How do you know if it is a half?

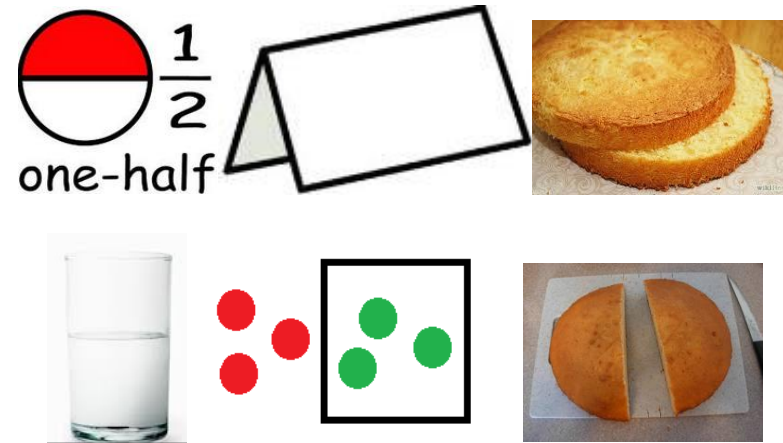
Fractions as operators

Objective: solve problems including halving

Concrete Experience: Find half of different objects (by folding, cutting, colouring and so on) and sets of objects. Draw lines half the size of another line.

Role play – shops, café, tea party (finding half a jug of drink, half a cake and so on).

Images:



Language: one half, halves, equal, two, equal parts, divide into 2, split, share, whole.

Symbols (when ready): $1/2$, $1 \div 2$

Questions: Show me half a page, half a ribbon, half of these six eggs.

Give me half of the pencils in the pot. Here is a set of 12 pencils. How many is half the set?



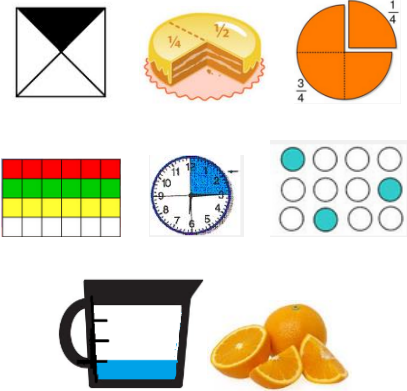
Year 1 Numbers – fractions

Understanding fractions

Objective: understand unit fractions ($\frac{1}{2}$ and $\frac{1}{4}$)

Concrete Experience: Ask children to cut a piece of paper into halves/quarters. Share objects into 4 equal groups. Build a tower a quarter of the size of the first. Create a pattern with $\frac{1}{4}$ of the objects one colour/shape and the rest different. Draw a line a quarter of the size of another. Mark a quarter on a number line. Complete quarter turns. Emphasise that quarters should be exactly the same size. Mark half on a number line and find halfway points on number lines, distances etc.

Images:



Language: one half, halves, one quarter, quarters, equal, two, four, parts

Symbols: $\frac{1}{4}$

Questions: How many halves/quarters make one whole? Leah says that this rectangle is divided into thirds because it is divided into three parts. Is she right? Explain your answer.



Counting in fractions

Objective: count in fractions ($\frac{1}{2}$ and $\frac{1}{4}$)

Concrete Experience: Start with real things, e.g. apples, toast.



Cut in half and/or quarters.



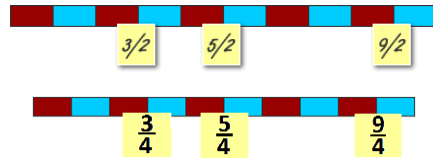
Count using the language they are familiar with – one half apple, two half apples, three half apples, four half apples etc.

Images: Use pictures to represent the things e.g. circles to represent the apples.



Drop the name of the object when counting, i.e. one half, two halves, three halves.

Match counting to a counting stick and placing numbers on it.



<http://www.topmarks.co.uk/Flash.aspx?f=EggFractions>

Language: fraction, quarter(s)

Symbols: $\frac{1}{2}$ $\frac{1}{4}$

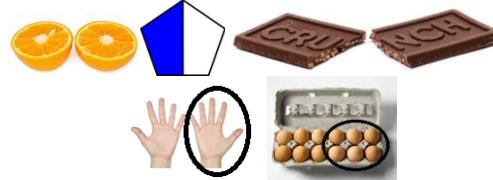
Questions: How many halves would I have if I cut 3 pieces of toast in half? Can you show me? How many oranges do I need to make 6 half oranges? How do you know?

Fractions as operators

Objective: recognise, find and name a half as one of two equal parts of an object, shape or quantity

Concrete Experience: Find half of different objects (by folding, cutting, colouring and so on) and sets of objects. Role play – shops, café, tea party (finding half a jug of drink, half a cake and so on).

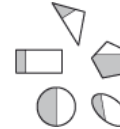
Images:



Language: one half, halves, equal, two, equal parts, divide into 2, split, share, whole.

Symbols: $\frac{1}{2}$, $1 \div 2$

Questions: How will you find half of that circle? How will you find half of these counters? Which shape is more than half shaded?



There are twenty children in a classroom. Half of them are girls. How many are boys? Explain how you worked it out. What number is halfway between 6 and 12? How did you work it out? How could we give someone half of 20p if we had one 20p coin? What about half of 12p if we had one 10p and two 1p coins? What is half of this amount?



Complete the shading on this diagram so that one half of the shape is shaded.

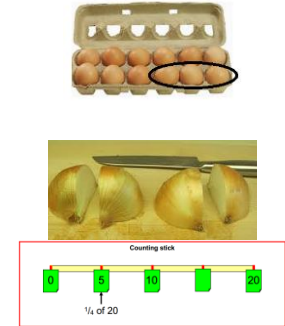


How could we work out half of three equal strips of paper?

Objective: recognise, find and name a quarter as one of four equal parts of an object, shape or quantity

Concrete Experience: Fold paper strips into quarters (move onto adding numbers). Fold paper with paint and make 4 identical images. Cut dough into 4 equal pieces. Share objects out equally into 4 groups.

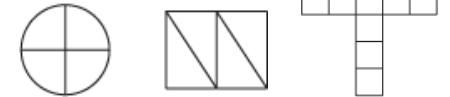
Images:



Language: one quarter, quarters, equal, four, equal parts, divide into 4, split, share, whole.

Symbols: $\frac{1}{4}$, $1 \div 4$

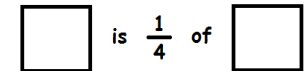
Questions: Shade one quarter of each shape.



In PE, can you turn through a quarter turn clockwise and anticlockwise? Now make a three quarter turn. How could you find one quarter of a piece of string? What about a quarter of two pieces of string? Here is a set of 12 pencils. How many is a quarter of the set?



Complete this statement in different ways:



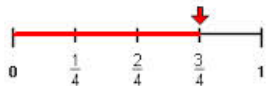
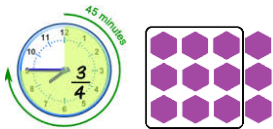
Year 2 Numbers – fractions

Understanding fractions

Objective: understand unit fractions ($\frac{1}{2}$, $\frac{1}{4}$) and non-unit fractions ($\frac{3}{4}$)

Concrete Experience: Cut items into quarters and select 3 of them. Share objects equally into 4 groups and select 3 of the groups. Build a tower three quarters the size of another. Turn $\frac{3}{4}$ turns. Mark $\frac{3}{4}$ onto a number line. Make patterns with $\frac{3}{4}$ being one colour/shape and the rest being another colour/shape.

Images:



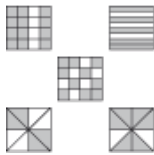
Language: numerator (tells you the 'number' of), denominator (nominates or names the fraction), one half, halves, one quarter, three quarters, equal parts, whole.

Symbols: $\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$

Questions: Complete the shading on this diagram so that $\frac{1}{2}$ is shaded. Describe the shaded part in another way.



Two of these shapes have three quarters shaded. Point to them. Explain how you know.



Explain how to find three quarters of a set of objects/a shape/ on a number line etc.

Counting in fractions

Objective: count in fractions ($\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$) up to 10, starting from any number and using the $\frac{1}{2}$ and $\frac{2}{4}$ equivalence on the number line (e.g. $1\frac{1}{4}$, $1\frac{2}{4}$ (or $1\frac{1}{2}$), $1\frac{3}{4}$, 2)

Concrete Experience: Start with real things, e.g. strips of plasticine.



Cut into fractions, e.g. thirds.



Count with name: a third of a strip, two thirds of a strip...

Images: Use pictures to represent objects.



Count without object name: one third, two thirds, three thirds, four thirds...



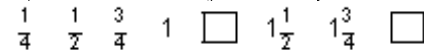
Count placing numbers on the counting stick. Count noticing the equivalence: one third, two thirds, one whole, one and one third, one and two thirds...

<http://www.topmarks.co.uk/Flash.aspx?f=EggFractions>

Language: third(s), equivalent, equivalence, equal to, same as

Symbols: $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $1\frac{1}{2}$, $1\frac{3}{4}$

Questions: How else could I say 7 halves? How do you know? How many halves are there in 3 whole apples and half an apple? Can you show me? What is a half more than three halves? What is 5 and a half subtract two halves? How many halves are there in two and a half? What is equivalent to five halves? What are the missing numbers in the sequence?



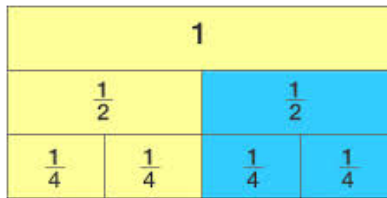
Equivalent fractions (percentages and decimals)

Objective: recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$

Concrete Experience: Find $\frac{1}{2}$ and then $\frac{2}{4}$ of a set of objects and compare amounts. Shade in $\frac{1}{2}$ then $\frac{2}{4}$ of shapes and compare the sizes. Use fraction cards to overlay $\frac{2}{4}$ onto $\frac{1}{2}$.

Images:

x	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15



Language: equivalent, fraction, balance, equal, whole, numerator, denominator

Symbols: $\frac{1}{2} = \frac{2}{4}$

Questions: Tell me some fractions that are equivalent to $\frac{1}{2}$ How do you know? Are there any others? How do you know when a fraction is equivalent to $\frac{1}{2}$?

How could you show that $\frac{1}{2}$ is equivalent to $\frac{2}{4}$?

Fractions as operators

Objective: recognise, find, name and write fractions $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity

Concrete Experience: Give children opportunities to find fractions of shapes where the shape is divided into small pieces. These should not always be regular. Cut, move, share objects that involve measures, e.g. string, dough, drinks etc.

Images:

Use models and images alongside oral work. For example, display 12 small objects such as counters.

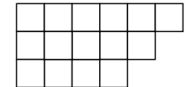
Ask questions such as:
 'What is one third of these 12 counters?'
 'What is two thirds of 12 counters?'
 'What is three thirds of 12?'

Arrange the counters in ways that help children to see the process and gradually reduce the reference to the counters as the children become more confident. Record the steps with the children and encourage them to recognise the underlying counting in 4s.

Language: fraction, equal parts, numerator, denominator, divide, division, multiply, multiplication

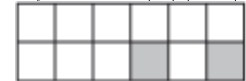
Symbols: $\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{3}$ $\frac{1}{5}$

Questions: Shade $\frac{1}{3}$ of this shape. How many squares is it made from? What is $\frac{1}{3}$ of 15? How do you know? How many squares do you need to shade?



Explain how we could find one quarter of this set of 12 pencils? What about three quarters?

Shade more squares so that exactly half of the shape is shaded.



Take 20 counters. Can you show me one quarter? Two quarters? Three quarters? Four quarters? What do you notice? Can you write that down in some way?

Here is a set of 12 pencils. How many is three quarters of the set?



Find three quarters of 20 biscuits. Three quarters of 24 buttons. How will you find one quarter of that rectangle? Three quarters? Here is a pizza cut into eight equal pieces. How many pieces are needed for three quarters of the pizza?

Take 20 cubes. Make a shape which is $\frac{1}{2}$ red and $\frac{1}{4}$ blue. What fraction of the shape is not red or blue?

How can you find $\frac{1}{3}$ of 27?

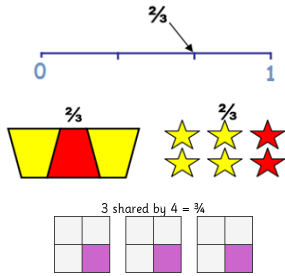
Year 3 Numbers – fractions

Understanding fractions

Objective: recognise fractions in the context of parts of a whole, numbers, measurements, a shape, and unit fractions as a division of a quantity

Concrete Experience: Cut items into fractions. Fold paper into different fractional pieces. Share objects equally into groups and select a number of the groups. Build a tower a fraction of the size of another. Turn in fractional turns. Mark fractions onto a number line. Make patterns with one fraction or multiple fractions being different colours/shapes.

Images:



Language: numerator, denominator, one half, halves, one quarter, three quarters, third, fifths tenths, equal, parts, whole.

Symbols:

$$8 \div 2 = \frac{8}{2} = 2\overline{)8} = \text{half of } 8$$

$$= 8 \times \frac{1}{2} = \text{how many 2s in } 8?$$

$$\frac{1}{5} \times 10 = \text{one fifth of } 10 = 10 \div 5$$

$$= 5\overline{)10} = \text{how many 5s in } 10?$$

Questions: What fraction of this shape is shaded? Could the same fraction be shaded in another way? How many?



What fraction of these tiles is circled?

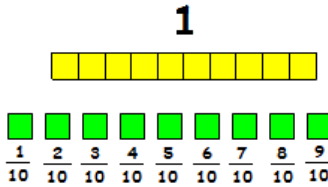


Tell me some fractions that are greater than $\frac{1}{2}$. How do you know?

Objective: recognise that tenths are dividing an object into 10 equal parts or dividing one-digit numbers or quantities by 10 (connect to place value, decimal measures and to division by 10)

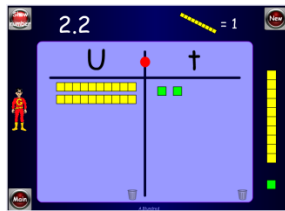
Concrete Experience: Ask children to make a strip of paper 1 metre long. Use a metre stick to help divide the strip into ten equal pieces.

Images: 'Blow-Up' Dienes so: Long = 1 and Unit = $\frac{1}{10} = 0.1$



<http://www.topmarks.co.uk/Flash.aspx?f=diennesandcoinsv3>

<http://www.topmarks.co.uk/Flash.aspx?f=diennesandcoinsv3>



Language: place value, partition, digit, ones, units, tens, hundreds, one-digit number, two-digit number, three-digit number, tenths

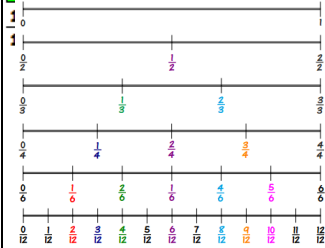
Symbols: $1 \div 10, \frac{1}{10}$

Questions: What fraction is each piece? How many tenths make one whole? What is one tenth less/more than...? How could you show a tenth?

Objective: recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators

Concrete Experience: Use knowledge of counting in fractions to place fractions along a number line and recognise their value. Use knowledge of tenths to find fractions of 10 and place them on a number line.

Images:



Language: place value, partition, digit, ones, units, tens, hundreds, one-digit number, two-digit number, three-digit number, tenths

Symbols: $\frac{1}{2}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{5}{5}, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}, \frac{10}{10}$

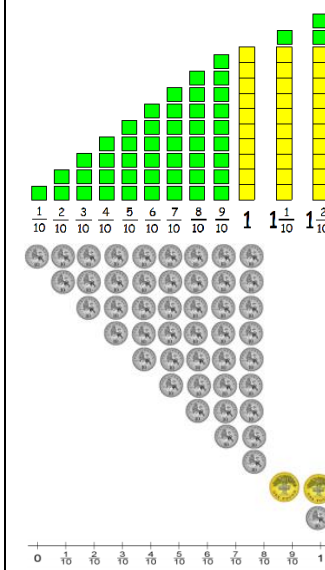
Questions: Draw an arrow on the number line to show $1\frac{3}{4}$. I ate more than $\frac{1}{2}$ a pizza but less than $\frac{3}{4}$. What fraction could I have eaten?

Counting in fractions

Objective: count up and down in tenths

Concrete Experience: Start with real things, e.g. strips of paper, or Dienes rods and units - 1 rod = 1 whole. 1 unit = 1 tenth, or money - £1 = 1 whole, 10p = 1 tenth. Count (using images) in tenths from zero, then from any whole number, any tenth and then any number (e.g. 2 and a half, 2 and 6 tenths...)

Images:



Language: tenths, numerator, denominator

Symbols: $\frac{1}{10}$

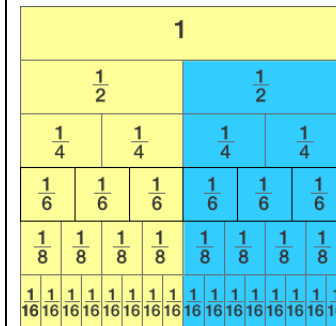
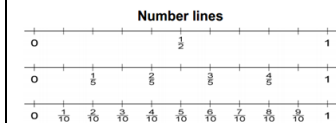
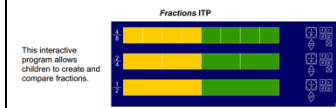
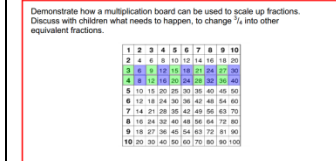
Questions: Why does the numerator change? If we count in tenths, will the denominator change? What does it mean if the numerator is bigger than the denominator/the denominator bigger than the numerator/the numerator and denominator are the same size?

Equivalent fractions (percentages and decimals)

Objective: recognise and show, using diagrams, equivalent fractions with small denominators

Concrete Experience: Use fraction cards and/or cuisiniere rods to make a fraction wall and explore equivalence.

Images:



Language: equivalent, fraction, balance, equal, whole, numerator, denominator

Symbols: $1 = \frac{3}{3}, \frac{1}{2} = \frac{2}{4}$ and so on.

Questions: Tell me some fractions that are equivalent to $\frac{1}{4}$. How do you know? Are there any others? What about $\frac{3}{4}$? How do you know that two fractions are equivalent?

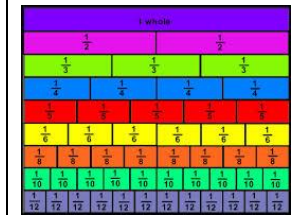
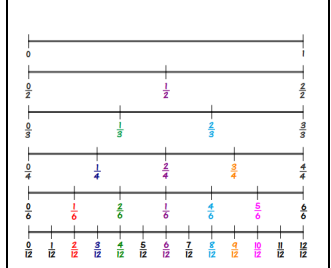
How could you show that $\frac{3}{6}$ is equivalent to $\frac{1}{2}$? What do you know about $\frac{3}{3}$ and $\frac{6}{6}$?

Objective: understand unit and non-unit fractions as numbers on the number line, and deduce relations between them, such as size and equivalence

Concrete Experience: Use knowledge of counting in fractions to place fractions along a number line and recognise their value. Compare the size of different fractions.

Use fraction cards and/or cuisiniere rods to make a fraction wall and compare fractions. Take equal strips of paper. Fold one into halves, one into quarters (half and half again) and one into eighths (half, half and half again). Label each half, quarter and eighth. Use this to discuss how many halves make one whole, how many quarters make one whole, etc.

Images:



Language: place value, partition, digit, ones, units, tens, hundreds, one-digit number, two-digit number, three-digit number, tenths, equivalent, fraction, balance, equal, whole, numerator, denominator

Symbols: $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \infty =$

Questions: Draw an arrow on the number line to show $\frac{3}{4}$.

I ate more than $\frac{1}{10}$ a cake but less than $\frac{1}{4}$. What fraction could I have eaten? Write a fraction that is larger than $\frac{1}{2}$.

Year 3 continued

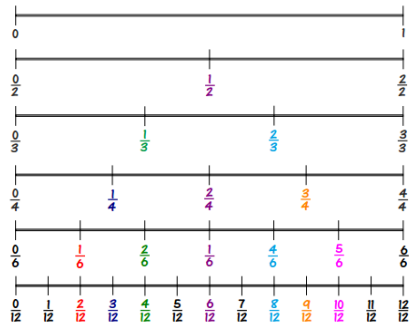
Compare, order and round

Objective: compare and order unit fractions, and fractions with the same denominators

Concrete Experience: 'Make' different fractions by folding/cutting paper, cutting items, colouring shapes, and so on. Compare the sizes of each and order them.

Use a fraction cards or number line to identify different fractions, and compare and order them.

Images:



Language: vinculum, numerator, denominator, division, equivalent, equal part of.

Symbols: $\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{1}{10}$ $\frac{3}{10}$ $\frac{7}{10}$ $\frac{9}{10}$ < >

Questions: Which is smaller – a quarter of an elephant or a quarter of a mouse?

Write a fraction that is larger than $\frac{2}{7}$.

What would you divide by if you needed to find $\frac{1}{2}$, $\frac{1}{4}$, etc?

Can you visualise fractions with the same denominator to help you order it?

Fractions as operators

Objective: recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators

Concrete Experience:

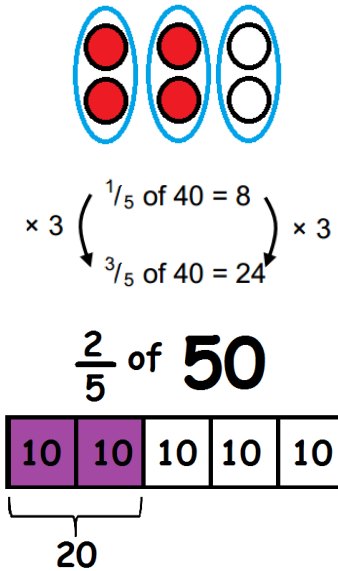
Give children practical experience of dividing shapes into fractions. For example:

Give children a rectangle that is 10 cm long and access to a ruler.

Explain that you want them to draw lines to divide the rectangle into fifths.

Ask: How many fifths make one whole? How many pieces must we divide the rectangle into? How can we make sure that each piece is the same size?

Images:



Language: fraction, part, equal parts, one whole, parts of a whole, number of parts, divide, one half, one third, one quarter, one fifth, one sixth, one tenth, unit fraction, numerator, denominator, multiply, multiple.

Symbols: $\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{1}{10}$ $\frac{3}{10}$ $\frac{7}{10}$ $\frac{9}{10}$

Questions: Would you rather have $\frac{1}{5}$ of 30 sweets or $\frac{3}{4}$ of 12 sweets? Why?

What would you prefer: 3 pizzas shared between 4 people or 6 pizzas shared between 10 people? Explain why.

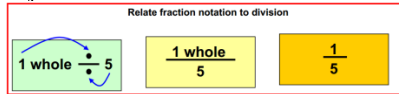
One third of a number is 7. What is the number?

What is $\frac{1}{3}$ of 9, 12, 15...? How did you work it out?

Objective: understand the relation between unit fractions as operators (fractions of), and division by integers

Concrete Experience: Share objects between children. Use Dienes to model processes.

Images:



A number line can then be used as a jotting/image to help solve fraction as operator problems. E.g. when finding $\frac{1}{3}$ of £60, drawing a line, then marking 0 and £60 at either end might help the children to connect finding a fraction (a third) to dividing ($\div 3$).

Language: fraction, part, equal parts, one whole, parts of a whole, number of parts, divide, one half, one third, one quarter, one fifth, one sixth, one tenth, unit fraction, numerator, denominator, multiply, multiple.

Symbols: $\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{1}{10}$ $\frac{3}{10}$ $\frac{7}{10}$ $\frac{9}{10}$

Questions: Tell me how to find one sixth of 42. What operation must you do to find one seventh of a number?

$\frac{1}{3}$ of 75 is 25. Write this as a division statement.

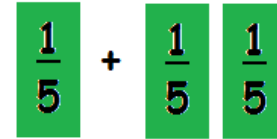
What operation would you key into a calculator to find $\frac{1}{8}$ of 256?

Calculating with fractions (decimals, and percentages)

Objective: add and subtract fractions with the same denominator within one whole [for example, $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$]

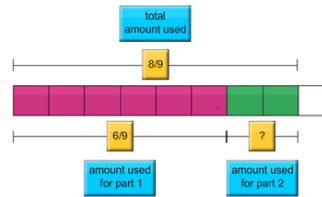
Concrete Experience: Use fraction cards to model the process.

Images:



Singapore Bar Method:

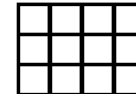
Clay is making an art project. He used $\frac{6}{9}$ meter of elastic for the first part. When he finished, Clay had used $\frac{8}{9}$ meter of elastic. How much elastic had Clay used in the second part of the project?



Language: fraction, part, equal parts, one whole, parts of a whole, number of parts, divide, one half, one third, one quarter, one fifth, one sixth, one tenth, unit fraction, numerator, denominator, add, subtract.

Symbols: $+$ $-$ $\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{1}{10}$ $\frac{3}{10}$ $\frac{7}{10}$ $\frac{9}{10}$

Questions: Use this 3 by 4 rectangle to find two fractions that add up to 1.



Here is a chocolate bar.



Bill eats 3 pieces and Ann eats 2 pieces. What fraction of the chocolate bar remains?

Solve problems

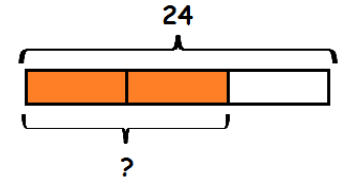
Objective: solve problems that involve all of the above

Concrete Experience: Use RUCSAC (or similar problem solving process) to: read, understand (and model), calculate, solve, answer and check. Select relevant models and images used before (selecting most appropriate) and/or Singapore Bar Method (below). Investigate statements about fractions (Always, Sometimes, Never).

Images: use a variety of models and images (previously shown) to solve problems.

Singapore Bar Method:

Kelly buys 24 flowers. Two thirds of them are white. How many white flowers are there?



Language: fraction, part, equal parts, one whole, parts of a whole, number of parts, divide, one half, one third, one quarter, one fifth, one sixth, one tenth, unit fraction, numerator, denominator, add, subtract, multiply, multiple, equivalent.

Symbols: $+$ $-$ \times \div $<$ $>$

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{1}{10}$ $\frac{3}{10}$ $\frac{7}{10}$ $\frac{9}{10}$

Questions: A half of an object is bigger than a quarter of an object. Always/Sometimes/ Never?

Fold a piece of paper in half. Now fold another piece in half in a different way. Are there any other ways? What is the same? What is different?

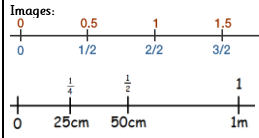
Halving a number less than 20 gives an answer of less than ten. Always/Sometimes/ Never?

Year 4 Numbers – fractions

Understanding fractions

Objective: use the number line to connect fractions, numbers and measures

Concrete Experience:
Use measuring equipment and measure whole quantities, halves, quarters and so on. Mark the quantities along a number line measured in fractions.



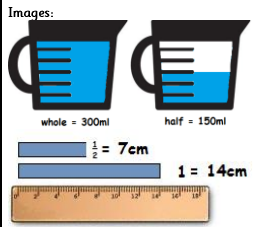
Language: measurements, one metre, one hundred centimetres, one litre, 1000 millilitres, half, quarter, three quarters, tenth, fifth, third

Symbols: m, cm, km, l, ml, kg, g, half, quarter, fifth, tenth, third,
 $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{10}$ $\frac{1}{100}$

Questions: If this jug holds 500ml, what would half of the jug be? Show me on a number line. Can you mark the divisions on this number line in tenths of the whole amount?

Objective: make connections between fractions of a length, of a shape and as a representation of one whole or set of quantities

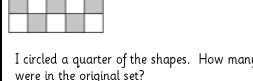
Concrete Experience:
Measure strips of paper then fold in half and measure again and so on.
Fill a jug with water and measure, calculate and pour out half and so on.



Language: measurements, one metre, one hundred centimetres, one litre, 1000 millilitres, half, quarter, three quarters, tenth, fifth, third

Symbols: m, cm, km, l, ml, kg, g, half, quarter, fifth, tenth, third,
 $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{10}$ $\frac{1}{100}$

Questions: What fraction of this shape is shaded? Can you say this fraction in another way?



Language: place value, partition, digit, ones, units, tens, hundreds, one-digit number, two-digit number, three-digit number, tenths, hundredths

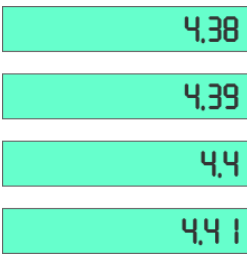
Symbols: $1 \div 10$, $1 \div 100$, $\frac{1}{10}$, $\frac{1}{100}$

Questions: What fraction is each piece? How many hundredths make one whole? What is one hundredth less/more than ...? How could you show a hundredth? Which of these decimals means $\frac{7}{10}$? A 70 B 7 C 0.7 D 0.07

Counting in fractions

Objective: count up and down in hundredths

Concrete Experience:
Review counting in tenths and relate counting in tenths to counting in decimals (0.1, 0.2, 0.3...) before moving onto hundredths. Start with real things, e.g. Dienes rods and units – 1 flat = 1 whole, 1 rod = 1 tenth. 1 unit = 1 hundred, or money – £1 = 1 whole, 10p = 1 tenth, 1p = 1 hundredth. Count (using images) in hundredths from zero, then from any whole number, any hundredth and then any number (e.g. 2 and a half, 2 and 51 hundredths.)
Make the calculator into a counting machine by using the 'constant' function, e.g. 4.38 + 0.01 = etc.



Images:

0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
1	2	3	4	5	6	7	8
10	20	30	40	50	60	70	80
100	200	300	400	500	600	700	800

For decimal numbers with up to two places, use a 10 x 10 grid so that each square represents 0.01 and each row represents 0.1. Discuss the effect of repeatedly adding the same decimal number, for example 0.01

Language: units, tenths, hundredths, numerator, denominator, equivalence.

Symbols: $\frac{1}{10} = 0.1$, $\frac{1}{100} = 0.01$

Questions: Which is greatest: one tenth or one hundredth? What is the next hundredth after 1.2? What is the hundredth before 3.5?

Equivalent fractions (percentages and decimals)

Objective: use factors and multiples to recognise equivalent fractions and simplify where appropriate

Concrete Experience:
Fill in multiplication tables to enable children to see the equivalence and simply fractions (the factor at the top of the column indicates the number to divide by to simplify the fraction) and solve problems, for example:

$$\frac{3}{4} = \frac{12}{16}$$

3	6	9	12
4	8	12	16

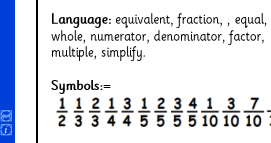
3	3	6	9	12
4	4	8	12	16

1	2	3	4	
3	3	6	9	12
4	4	8	12	16

1	2	3	4	
3	3	6	9	12
4	4	8	12	16

Calculate $16 \div 4$ work back in multiples of that amount.

Images:



Language: equivalent, fraction, equal, whole, numerator, denominator, factor, multiple, simplify.

Symbols: =
 $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{10}$ $\frac{1}{100}$

Questions: What is the missing number?
 $\frac{7}{10} = \frac{\square}{30}$ How do you know?

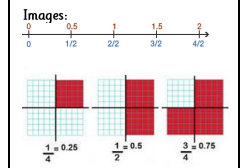
Tell me a fraction that is equivalent to but has a denominator of 9. How did you do it?
 Find the missing number
 $\frac{3}{\square} = \frac{12}{16}$

Karen makes a fraction using two number cards. She says,
 'My fraction is equivalent to $\frac{1}{2}$. One of the number cards is 6'

What could Karen's fraction be?
 Give both possible answers.
 What clues did you look for to cancel these fractions to their simplest form?
 How do you know when you have the simplest form of a fraction?

Objective: recognise and write decimal equivalents to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$

Concrete Experience:
Link to counting in fractions. Use a simple number line marked in divisions of 0.5 to familiarise children with counting forwards and backwards in steps of 0.5. Extend this to other number lines to develop counting in other step sizes (e.g. 0.2).
Use 10x10 grids and establish each square is one hundredth (0.01). Find fractions of the square (100) and use it to write decimal equivalents.
Use a calculator to carry out the division, e.g. $\frac{3}{4}$ would be $3 \div 4$.



Language: numerator, denominator, equivalent, proper fraction, decimal fraction, decimal place, decimal point

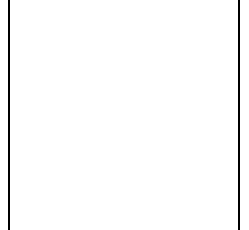
Symbols: $\div = \frac{1}{2}$, 0.5

Questions: What is three quarters as a decimal? How would you find the decimal equivalent of $\frac{1}{4}$?

Symbols: $1 \div 10$, $1 \div 100$, $\frac{1}{10}$, $\frac{1}{100}$

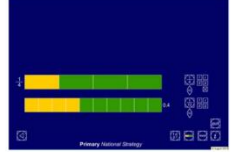
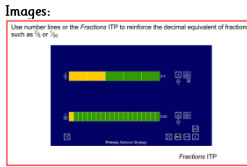
Questions: Tell me two fractions that are the same as 0.5. Are there any other possibilities? How many pence are the same as £0.25? How many hundredths are the same as 0.75m? How else could you write twenty-five hundredths? How many centimetres are the same as 0.75m? How else could you write seventy-five hundredths?
 Which of these fractions is the same as 0.5 ? $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$
 Which of these is the same as 0.4? A four B four tenths C four hundredths D one fourth
 Which of these fractions is the same as nought point four?
 $\frac{1}{4}$ $\frac{1}{40}$ $\frac{1}{400}$ $\frac{4}{10}$ $\frac{4}{100}$

How would you write $\frac{37}{100}$ as a decimal?
 Which number represents the shaded part of the figure?



Objective: recognise and write decimal equivalents of any number of tenths or hundredths

Concrete Experience:
Establish that the decimal point is used to separate whole amounts and parts of the whole.
Use a calculator and the language of fractions to find decimal and fraction equivalents.
Present children with commonly confused fraction and decimal equivalents, for example, 0.4 and $\frac{1}{4}$.



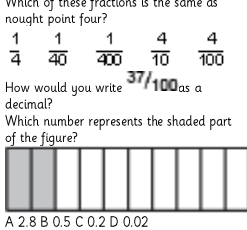
Language: place value, decimal, tenths, hundredths, 3.6 = three point six = three units and six tenths
 0.13 = one tenth + three hundredths = 13 hundredths.

Questions: Tell me two fractions that are the same as 0.5. Are there any other possibilities? How many pence are the same as £0.25? How many hundredths are the same as 0.75m? How else could you write twenty-five hundredths? How many centimetres are the same as 0.75m? How else could you write seventy-five hundredths?
 Which of these fractions is the same as 0.5 ? $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$
 Which of these is the same as 0.4? A four B four tenths C four hundredths D one fourth
 Which of these fractions is the same as nought point four?
 $\frac{1}{4}$ $\frac{1}{40}$ $\frac{1}{400}$ $\frac{4}{10}$ $\frac{4}{100}$

Symbols: $1 \div 10$, $1 \div 100$, $\frac{1}{10}$, $\frac{1}{100}$

Questions: Tell me two fractions that are the same as 0.5. Are there any other possibilities? How many pence are the same as £0.25? How many hundredths are the same as 0.75m? How else could you write twenty-five hundredths? How many centimetres are the same as 0.75m? How else could you write seventy-five hundredths?
 Which of these fractions is the same as 0.5 ? $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$
 Which of these is the same as 0.4? A four B four tenths C four hundredths D one fourth
 Which of these fractions is the same as nought point four?
 $\frac{1}{4}$ $\frac{1}{40}$ $\frac{1}{400}$ $\frac{4}{10}$ $\frac{4}{100}$

How would you write $\frac{37}{100}$ as a decimal?
 Which number represents the shaded part of the figure?



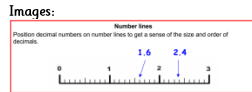
A 2.8 B 0.5 C 0.2 D 0.02

Year 4 - continued

Compare, order and round

Objective: compare numbers with the same number of decimal places up to two decimal places

Concrete Experience:
Help children become aware of the relative size of decimal numbers by ordering a set of amounts of money or lengths. Link this to ordering numbers on a number line. Make sure that you include numbers to overcome misconceptions such as mistaking the length of the number with its size, for example thinking that 4.05 is larger than 4.5.



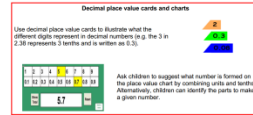
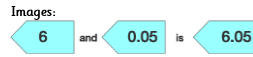
Language: decimal, equal parts of, decimal point, equivalent, place value, units, tenths, hundredths.

Symbols: 0.1

Questions:
Which is the larger amount, £0.75 or 90p? Which is longer, 3.06 m or 3.6 m? Which is larger: 239p or £2.93? Why?

Objective: understand the place value of decimals

Concrete Experience:
Use arrow cards. These can be used to build and undo decimal numbers e.g. 6.05 is 6 ones, and 5 hundredths. When modelling numbers are put together you could say
What is 6 + 0.05
What is 0.05 more than 6
What is 6 subtract 0.05
What does the 5 represent? How else could we say it or write it?



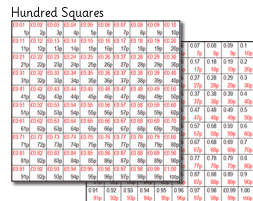
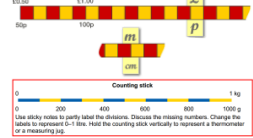
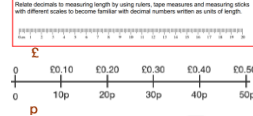
Language: place value, units, decimal point, tenths, hundredths

Symbols: 6.05 < > =

Questions: Can you tell me what the digit 7 represents in each of these amounts: £2.70, 7.35 m, £0.37, 7.07 m? Which is larger: 239 cm or 2.93 m? Why?

Objective: represent numbers with 1 or 2 d.p. in several ways, such as on number lines

Concrete Experience:
Model decimal in numbers in various ways (see below) and ask children to do the same. Present children with mistakes and ask them to identify the mistake and correct them.



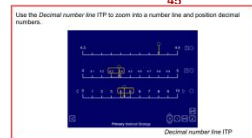
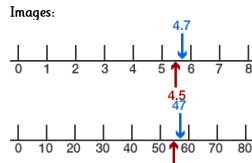
Language: place value, units, decimal point, tenths, hundredths 3.6 = three point six = three units and six tenths
0.13 = one tenth + three hundredths = 13 hundredths.

Symbols: 7.23

Questions: Write down a number lying between 7 and 8. Write it as a fraction and then as a decimal. Enter 5.3 on to your calculator display. How can you change this to 5.9 in one step (operation)? A CD costs between £5.50 and £5.65. How much could it cost?

Objective: round decimals with one decimal place to the nearest whole number

Concrete Experience:
Mark number on a number line. Identify the whole numbers surrounding it, and mark the midpoint between the two whole numbers. This will clearly indicate the whole number to round to.
Compare to rounding 2 digit numbers to the nearest 10.



Language: place value, ten, unit, tenths, halfway, nearest, round.

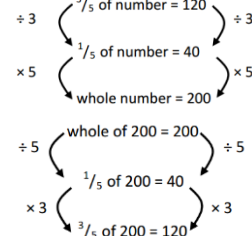
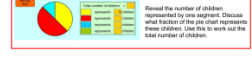
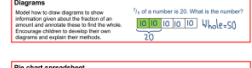
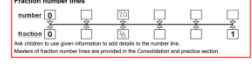
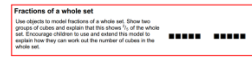
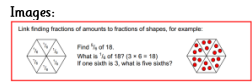
Symbols:

Questions: What is 4.7 rounded to the nearest whole number? I rounded my number to 3. What number (with one decimal place) could it have been? What is the biggest/smallest number I would round to 2? Simon rounded 1.6 to 2. Was he right? Explain how you know. Show me why I would round 2.3 to 2. My chair is nearly 1 m high. How tall could it be?

Fractions as operators

Objective: recognise, find and write fractions of a discrete set of objects:

Concrete Experience:
Use different models and images to help children understand that a fraction such as 4/5 is 1/5 x 4, and finding 1/5 is the first step to finding 4/5. This will help them begin to associate finding 4/5 with 'divide the whole into five equal parts and then group together four of these equal parts'.



Language: numerator, denominator, multiply, divide, whole, fraction.

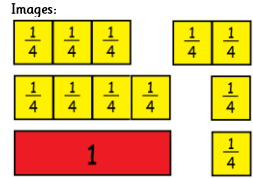
Symbols: x ÷ = 1/2 1/4 3/4

Questions:
Explain how you would solve this problem: There are 24 coloured cubes in a box. Three quarters of the cubes are red, four of the cubes are blue and the rest are green. How many green cubes are in the box? One more blue cube is put into the box. What fraction of the cubes in the box is blue now?
Three-quarters of a number is 48. What is the number?

Calculating with fractions (decimals, and percentages)

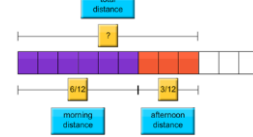
Objective: add and subtract fractions with the same denominator

Concrete Experience: Use fraction cards to model the process (including equivalence where needed, 1 e 4/4 = 1)



Singapore Bar Method:

Mr. Guzman drove 6/12 of the distance to Mount Prime in the morning and 3/12 of the distance in the afternoon. What fraction of the distance did Mr. Guzman travel?



<http://www.mathplayground.com/thinkingblocks.html>

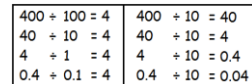
Language: fraction, part, equal parts, one whole, parts of a whole, number of parts, divide, one half, one third, one quarter, one fifth, one sixth, one tenth, unit fraction, numerator, denominator, add, subtract.

Symbols: + -
1 1 2 1 3 1 2 3 4 1 3 7 9
2 3 3 4 4 5 5 5 10 10 10 10

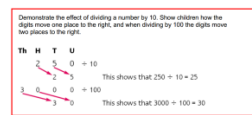
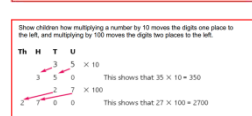
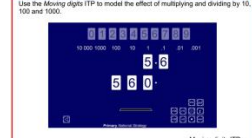
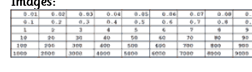
Questions:
What is 3/4 + 3/4? What is 1/2 - 1/2? What is 1/4 less than 1? How do you know? Show me how you worked it out.

Objective: find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths

Concrete Experience:
Use place value grids (tens, units, decimal point, tenths, hundredths, ...). These could be drawn on paper or made into a wall display for children to hold digit cards against. Create numbers and look at effect of dividing by 10 and 100. Discuss. Explore the visual pattern of the digits within division.



Link converting units of measure to estimating, in order to help children spot where they have made mistakes, for example, '1.5 m cannot equal 15 cm because I know 1.5 m is about my height but 15 cm is half the length of a ruler.'



Language: digit, decimal, multiply, times, divide, share, scale up, scale down, increase, decrease, factor, tens of thousands, thousands, hundreds, tens, units, ones, tenths, hundredths, thousandths

Symbols: +

Questions: What is ... ÷ 10/+ 100? How do you know?

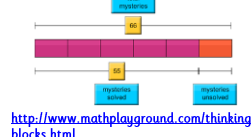
Solve problems

Objective: solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions with a whole number answer

Concrete Experience: Use RUCSAC (or similar problem solving process) to: read, understand (and model), calculate, solve, answer and check. Select relevant models and images used before (selecting most appropriate) and/or Singapore Bar Method (below). Investigate statements about fractions (Always, Sometimes, Never).

Images: use a variety of models and images (previously shown) to solve problems. Singapore Bar Method

The Snoop and Spy Detective Agency was hired to solve 66 mysteries last year. The detectives were able to solve 2/3 of the mysteries. How many mysteries did the detective agency solve?



<http://www.mathplayground.com/thinkingblocks.html>



<http://www.mathplayground.com/thinkingblocks.html>

Language: All those used before.
Symbols: All those used before.
Questions: Range of questions (see those above), including Always, Sometimes, Never statements etc.

Objective: solve simple measure and money problems involving fractions and decimals to two d.p.

Concrete Experience: Use RUCSAC (or similar problem solving process) to: read, understand (and model), calculate, solve, answer and check. Select relevant models and images used before (selecting most appropriate) and/or Singapore Bar Method (below). Investigate statements about fractions (Always, Sometimes, Never). See Calculation Policy.

Images: All those used before.
Language: All those used before.
Symbols: All those used before.

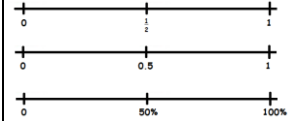
Year 5 Numbers – fractions

Understanding fractions

Objective: understand that percentages, decimals and fractions are different ways of expressing proportions

Concrete Experience: Find fractions of 100%. Calculate $1 \div 2$. Use strips of paper as number lines and fold.

Images:



Language: equivalence, fractions, percentages, decimals, division

Symbols: $\frac{3}{4} = 3 \div 4$

Questions:

Fill in the missing numbers in the grid

Fraction	Decimal	Percentage
$\frac{1}{2}$		
	0.2	
		75%
$\frac{3}{4}$		
	0.25	

Objective: recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal

Concrete Experience: Use money to show how 10p can be expressed as a percentage and a fraction of £1. Give children the opportunity to use coins to convince themselves that, for example, 10p is 1/10 or 10% of £1 because they need ten 10p coins to make £1.

Images:



Represent a percentage, using practical resources such as money (£1, 10p and 1p coins) or images, for example, a 10 by 10 square grid or the Area ITP.

Area ITP used to represent 35 shaded squares out of 100, or 35%.

Links can be made between fractions and percentages, using the Fractions ITP. This can help children realise that finding 50% is the same as halving, so that 25% they are finding one quarter, etc.

Fractions ITP

Language: percentage, per cent, tenths, hundredths, equivalent.

Symbols: %, 0.1, 0.01

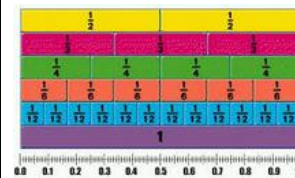
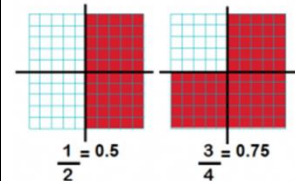
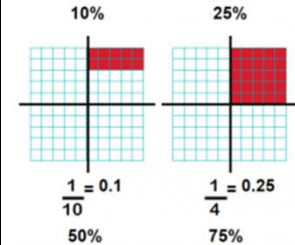
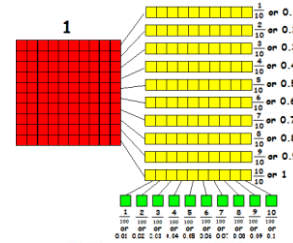
Questions: How can you model a percentage? How can it be written as a fraction with a denominator of 100? What would the equivalent decimal be?

Equivalent fractions (percentages and decimals)

Objective: read and write decimal numbers as fractions (for example, $0.71 = \frac{71}{100}$)

Concrete Experience: Use Dienes to represent decimal numbers and compare to Dienes as fractions. Use a 10 x 10 grid to represent 1. Colour in the decimal amount shown and compare to fractions of a 10x10 square. Use skills of simplifying form. Year 4 to simplify fractions where possible.

Images:



Language: equivalence, decimal fraction, decimal, fraction, equal

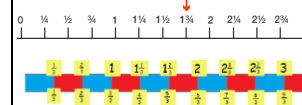
Symbols: 0.1, 0.01,

Questions: How could you write this fraction as a decimal? How could you write this decimal as a fraction with a denominator of 100? Can you simplify this fraction?

Objective: recognise mixed numbers and improper fractions and convert from one form to the other

Concrete Experience: Use knowledge of counting to mark numbers on number lines. Make fractions using fraction cards, exchanging equivalent units and so on.

Images:



Language: denominator, numerator, whole, fraction, equivalent, mixed number, improper fraction.

$$\frac{7}{5} = 1 \frac{2}{5}$$

Questions:

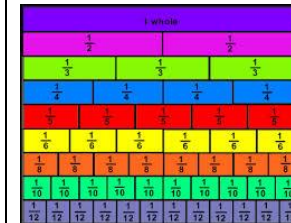
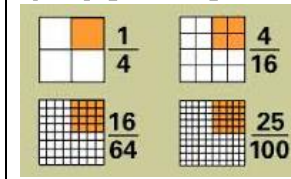
Tell me a fraction that is bigger than 3. How else could we write it? Show me what eight thirds looks like. What is equivalent to it?

Objective: identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths

Concrete Experience: Use various models and images to represent fractions in different ways. Compare fractions and identify equivalence. Investigate using multiplication and division to create equivalent fractions and to simplify fractions to find simplest equivalent.

Images:

x	1	2	3	4	5	6	7	8	9
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54



Language: numerator, denominator, decimal point, tenths, hundredths

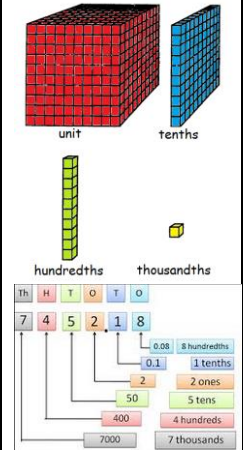
Symbols: =

Questions: Show me what... looks like. How else can we model...? What fraction is equivalent to...? How do you know if fractions are equivalent? How can we use multiplication/division to find equivalent fractions?

Objective: recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents

Concrete Experience: Use a calculator and the language of fractions to find decimal and fraction equivalents. 'Blow up' Dienes so 1 cube represents 1 whole, and therefore unit cubes represent one thousandths.

Images:



Language: numerator, denominator, decimal point, tenths, hundredths, thousandths.

$3.652 =$ 'three point six five = three units, six tenths, five hundredths and two thousandths = 3652 thousandths

Symbols: $1 \div 10$, $1 \div 100$, $1 \div 1000$

Questions: How would you read this number 1.234. What is the place value of each digit? How many units are there? How many tenths? How many hundredths? How many thousandths?

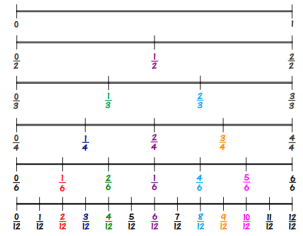
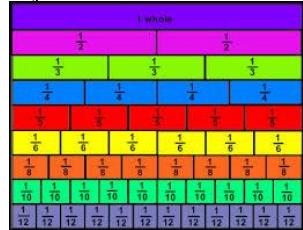
Compare, order and round

Objective: compare and order fractions with denominators that all have multiples of the same number

Concrete Experience:

Use fraction cards or a fraction wall to physically compare the sizes of fractions and place them on a number line.
Use knowledge of simplifying to compare fractions with denominators that all have multiples of the same number.
Use a calculator to perform the division and compare the decimal numbers, then relate them to the original fraction.

Images:



Language: multiple, numerator, denominator, equivalent, order.

Symbols:

$$\frac{1}{2} \frac{1}{3} \frac{2}{3} \frac{1}{4} \frac{3}{4} \frac{1}{5} \frac{2}{5} \frac{3}{5} \frac{4}{5} \frac{1}{10} \frac{3}{10} \frac{7}{10} \frac{9}{10}$$

< >

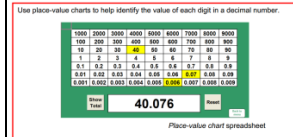
Questions: How can you tell how many of each fraction make one whole? (The denominator tells you.) Which is bigger, 1/4 or 1/8? Which do you think would be bigger, 1/50 or 1/100? How do you know?

Mark $\frac{1}{3}$ and $\frac{5}{6}$ on a number line.
What do the fractions $\frac{6}{9}$, $\frac{14}{21}$ and $\frac{18}{27}$ have in common?

Objective: read, write, order and compare numbers with up to three decimal places

Concrete Experience: consolidate understanding to read, write, order and compare numbers with up to two decimal places (Year 4) and extend to thousandths!
Link to ordering numbers in the thousands.

Images:



Language: decimal, decimal fraction, decimal point, decimal place, tenth, hundredth, thousandth, significant digit

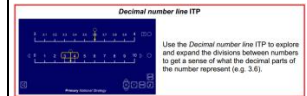
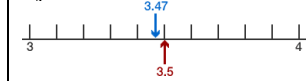
Symbols: < = >

Questions: Put these in order: £0.56, 125p, £3.60, 250p, 7p, £5, 205p. Which is the smallest? How do you know? Which is the largest? How do you know? What amount of money comes next: £1.76, £1.86, £1.96,...? What length comes next: 1.76 m, 1.86 m, 1.96 m, ...?

Objective: round decimals with two decimal places to the nearest whole number and to one decimal place

Concrete Experience:
Mark number on a number line. Identify the whole numbers surrounding it, and mark the midpoint between the two whole numbers. This will clearly indicate the whole number to round to. When rounding 3.47 to the nearest whole number, children need to be aware that the answer will either be 3 or 4 and this can clearly be seen on the number line below.
They then need to mark the middle point between 3 and 4 and know that it is 3.5. Marking on 3.47 allows children to see clearly that 3.47 is less than 3.5 and thus closer to 3 than 4.
Compare to rounding 3 digit numbers to the nearest 100. In this example it is also useful to make the comparison with rounding 347 to the nearest 100. Repeat but mark the surrounding tenths.

Images:



Language: place value, ten, unit, tenths, hundredths, halfway, nearest, round.

Symbols:

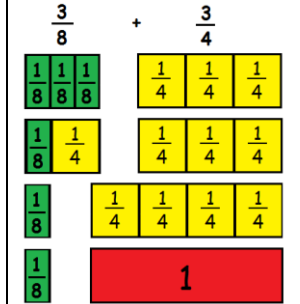
Questions: What is 4.37 rounded to the nearest whole number? I rounded my number to 5. What number (with two decimal places) could it have been? What is the biggest/smallest number I would round to 2.2? Simon rounded 1.64 to 2. Was he right? Explain how you know. Show me why I would round 2.3 to 2. I am nearly 1.6 m tall. How tall could I be?

Calculating with fractions (decimals, and percentages)

Objective: add and subtract fractions with the same denominator and denominators that are multiples of the same number and write mathematical statements > 1 as a mixed number (e.g. $\frac{2}{5} + \frac{3}{5} = \frac{6}{5} = 1 \frac{1}{5}$)

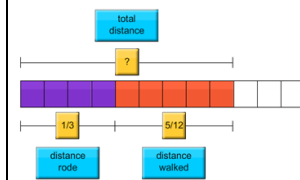
Concrete Experience: Use fraction cards to model the process (including equivalence where needed, $1 \frac{4}{4} = 1$).

Images:



Singapore Bar Method:

Alex traveled from his house to the tennis court. He rode 1/3 mile and then walked 5/12 mile. How far did Alex travel from his house to the tennis court?



Language: fraction, part, equal parts, one whole, parts of a whole, number of parts, divide, one half, one third, one quarter, one fifth, one sixth, one tenth, unit fraction, non-unit fraction, numerator, denominator, simplify, equivalence, add, subtract.

Symbols: + -

$$\frac{1}{2} \frac{1}{3} \frac{2}{3} \frac{1}{4} \frac{3}{4} \frac{1}{5} \frac{2}{5} \frac{3}{5} \frac{4}{5} \frac{1}{10} \frac{3}{10} \frac{7}{10} \frac{9}{10} \text{ etc}$$

Questions:

What is $\frac{3}{4} + \frac{3}{4}$? What is $\frac{4}{5} - \frac{3}{10}$?
What is $\frac{3}{4}$ less than $1\frac{1}{2}$? How do you know? Show me how you worked it out.

Solve problems

Objective: solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$ and those fractions with a denominator of a multiple of 10 or 25.

Concrete Experience: Use previous knowledge of equivalence of fractions, decimals and percentages. Use RUCSAC (or similar problem solving process) to read, understand (and model), calculate, solve, answer and check. Select relevant models and images used before (selecting most appropriate) and/or Singapore Bar Method (below). Investigate statements about fractions (Always, Sometimes, Never).
See Calculation Policy.

Images: All those used before.

Language: All those used before.

Symbols: All those used before.

Questions: Range of questions (see those above), including Always, Sometimes, Never statements etc.

Objective: solve problems involving number up to three decimal places

Concrete Experience: Use RUCSAC (or similar problem solving process) to read, understand (and model), calculate, solve, answer and check. Select relevant models and images used before (selecting most appropriate) and/or Singapore Bar Method (below). Investigate statements about fractions (Always, Sometimes, Never).
See Calculation Policy.

Images: All those used before.

Language: All those used before.

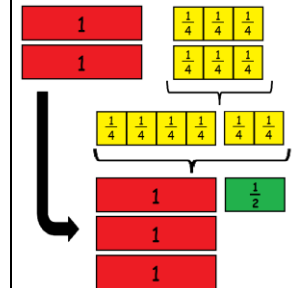
Symbols: All those used before.

Questions: Range of questions (see those above), including Always, Sometimes, Never statements etc.

Objective: multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

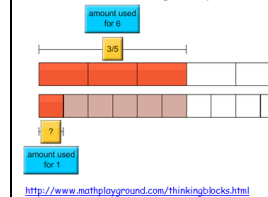
Concrete Experience: Use fraction cards to model the process (including equivalence where needed, $1 \frac{4}{4} = 1$)

Images: $1 \frac{3}{4} \times 2$



Singapore Bar Method:

Chef Ginnamon uses $\frac{3}{5}$ cup of oil to make 6 giant cup cakes. How much oil would Chef Ginnamon need to make 1 giant cup cake?



Language: fraction, part, equal parts, one whole, parts of a whole, number of parts, divide, one half, one third, one quarter, one fifth, one sixth, one tenth, unit fraction, non-unit fraction, numerator, denominator, simplify, equivalence, multiply.

Symbols: x

$$\frac{1}{2} \frac{1}{3} \frac{2}{3} \frac{1}{4} \frac{3}{4} \frac{1}{5} \frac{2}{5} \frac{3}{5} \frac{4}{5} \frac{1}{10} \frac{3}{10} \frac{7}{10} \frac{9}{10} \text{ etc}$$

Questions:

What is $1 \frac{3}{4} \times 2$? How can you show me? How would you explain to someone else how to do it? Is the answer to $4 \times 1 \frac{1}{4}$, 6? How do you know?

Year 6 Numbers – fractions

Understanding fractions

Objective: use a variety of images to support their understanding of multiplication with fractions (fractions as operators (fractions of), as numbers, and as equal parts of objects, for example as parts of a rectangle)

Concrete Experience: In $\frac{3}{5}$, the 3 tells you that you have three pieces and the 5 tells you that each piece is one fifth of a whole. One way to read $\frac{3}{5}$ is '3 in every 5'.

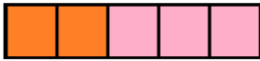
Images:



% of £20 = £15

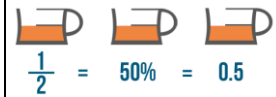


2:3

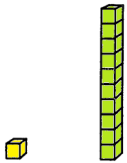


$\frac{2}{5}$

$\frac{3}{5}$



$\frac{1}{2} = 50\% = 0.5$



A tower one tenth of the size of the other.

Language: fraction, part, equal parts, one whole, parts of a whole, number of parts, divide, one half, one third, one quarter, one fifth, one sixth, one tenth, unit fraction, non-unit fraction, numerator, denominator, of, equals, multiply

Symbols: \times $+$ $-$ $/$:

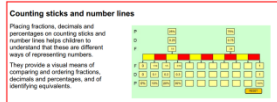
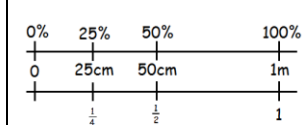
Questions: How many different ways can you show me...? Can you explain what this shows us? How would you explain...? Does this show ...? Why/why not?

Equivalent fractions (percentages and decimals)

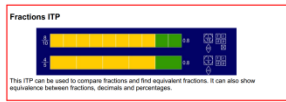
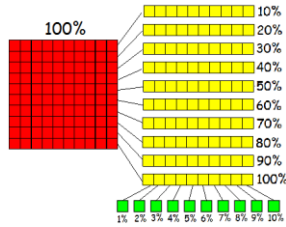
Objective: recall and use equivalences between simple fractions, decimals and percentages, including in different contexts

Concrete Experience: Using number lines is an effective way to see the equivalence between the numbers. This could be linked to measure, for example, finding 50% of 1m = $\frac{50}{100}$ of 100cm = $(100 \div 100) \times 50 = 50$ cm.

Images:



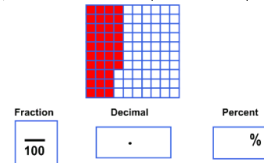
Use Dienes to represent percentages, e.g. 1 flat = 100% and so on.



Language: equivalence, decimal, percentage, fraction, equal.

Symbols: % =

Questions: Fill in the missing numbers in the grid



How do you know that 25% is the same as a quarter and the same as 0.25? How can you show me?

Objective: use common factors to simplify fractions; use common multiples to express fractions in the same denominator

Concrete Experience: Use knowledge of times table facts to find common factors, and division to simplify. Use multiplication to express fractions in the same denominator.

Images:

x	1	2	3	4	5	6	7	8	9
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54

$$\frac{3}{5} + \frac{2}{3}$$

$$\times 3 \quad \times 5$$

$$\frac{9}{15} + \frac{10}{15}$$

Language: numerator, denominator, fraction, proper/improper fraction, equivalent, reduced to, cancel, equal, whole, numerator, denominator, factor, multiple, simplify.

Symbols: \times \div =

Questions: What is the missing number?

$$\frac{7}{10} = \frac{\square}{30}$$

How do you know?

Tell me a fraction that is equivalent to $\frac{2}{3}$ but has a denominator of 9. How did you do it? Find the missing number

$$3 = \frac{12}{\square}$$

Karen makes a fraction using two number cards. She says,

'My fraction is equivalent to $\frac{1}{2}$. One of the number cards is 6'

What could Karen's fraction be? Give both possible answers. What clues did you look for to cancel these fractions to their simplest form? How do you know when you have the simplest form of a fraction?

Objective: associate a fraction with division and calculate decimal fraction equivalents (for example, 0.375) for a simple fraction (for example, $\frac{3}{8}$)

Concrete Experience: Link to performing division calculations (with decimal remainders).

Images:

Example: $\frac{3}{4} \rightarrow 3 \div 4$

$$4 \overline{)3.00}$$

$$\underline{-28}$$

$$20$$

$$\underline{-20}$$

$$0$$

$\frac{3}{4} = 0.75$

Language: equivalence, decimal fraction, decimal, fraction, equal

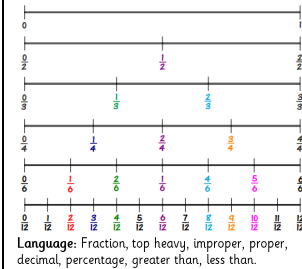
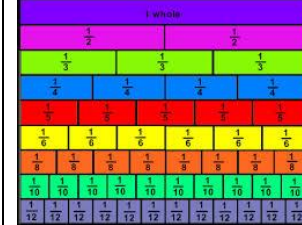
Symbols: \div =

Questions: Can you find the decimal fraction for $\frac{1}{2}$? Is the decimal equivalent of $\frac{3}{5}$ 2.5? How do you know? Can you show me..?

Objective: compare and order fractions, including fractions > 1

Concrete Experience: Label the fractions on a fraction wall or use the Fractions ITP to create strips that are divided into halves, thirds, quarters, etc. Compare the size of fractions and position them on a number line. Use knowledge of decimal and percentage equivalence to help position fractions.

Images:



Symbols: $<$ $=$ $>$

Questions: Which is bigger, $\frac{1}{2}$ or $\frac{1}{3}$? How do you know? Which is larger: $\frac{1}{3}$ or $\frac{2}{5}$? Explain how you know.

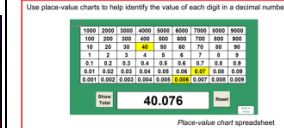
Arrange these numbers in order: 1, $\frac{3}{4}$, $\frac{15}{8}$, 1.6 - with a calculator and without a calculator. Which way of working do you prefer? Why?

Compare, order and round

Objective: identify the value of each digit in numbers to 3 d.p.

Concrete Experience: Build on knowledge of place value of decimal numbers (Year 4) and of thousandths (Year 5). Use place value charts and arrow cards to determine the value of each digit.

Images:



Language: place value, units, decimal point, tenths, hundredths, thousandths.

Symbols: =

Questions: Can you tell me what the digit x represents in each of these amounts...? Which is larger: x mm or y m? Why? How do you know? Show me.

Objective: develop their skills of rounding and estimating as a means of predicting and checking the order of magnitude of their answers to decimal calculations

Concrete Experience: Use rounding skills (from Year 5) to find estimates to calculations. Then perform calculations and compare. Provide children with incorrect answers to calculations and ask children to identify them by rounding and estimating.

Images:



Language: place value, ten, unit, tenths, hundredths, halfway, nearest, round, estimate, reasonable, approximate.

Symbols: \approx

Questions: What will be the approximate answer to ...? How did you work it out? Which is the best approximation to ...? Why?

Fractions as operators

Objective: use written division methods in cases where the answer has up to two decimal places

Concrete Experience: Set problems in a context – money/shopping, measures and so on.

Images: Also see Calculation Policy. I earn £186 in a 5 day week. How much do I earn in 3 days? Or What is $\frac{3}{5}$ of my weekly £186?

$$\begin{array}{r} 0.3720 \\ 5 \overline{) 1.8360} \\ \underline{5 } \\ 18 \\ \underline{15} \\ 36 \\ \underline{35} \\ 10 \\ \underline{10} \\ 0 \end{array}$$

$$\begin{array}{r} 37.20 \\ 3 \times \\ \underline{111.60} \\ 2 \end{array}$$

Language: Multiply, divide, numerator, denominator, decimal, decimal point, remainders.

Symbols: \times +

Questions: Tim has spent £3, which is $\frac{4}{5}$ of his pocket money. How much did he have?
Over 3 days Jo earns £186. How much money does she earn in a week?
Harry weighs $\frac{3}{8}$ of the weight of a Shetland pony that is 264kg. How much does Harry weigh?

Objective: use understanding of the relationship between unit fractions and division to multiply a quantity representing a unit fraction to find the whole quantity (for example, if $\frac{1}{4}$ of a length is 36cm, then the whole length is $36 \times 4 = 144$ cm)

Concrete Experience:

Model how to record the steps in a multistep problem so that each stage is clear. Encourage children to develop confidence by writing down every calculation they do, even when they work them out mentally or on a calculator.

For example: Charlie has saved £15 towards buying a computer game. This is $\frac{3}{5}$ of the cost of the game. How much does the game cost?
o We know that $\frac{3}{5}$ of the cost = £15
o So $\frac{1}{5}$ of the cost = £15 \div 3 = £5
o If $\frac{1}{5}$ of the cost is £5, then the whole cost = £5 \times 5 = £25
o The game costs £25

Images:

Language: Multiply, divide, numerator, denominator, decimal, decimal point, remainders, estimate.

Symbols: \times +

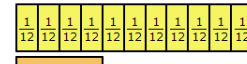
Questions: Charlie has saved £15 towards buying a computer game. This is $\frac{3}{5}$ of the cost of the game. How much does the game cost?
If $\frac{1}{4}$ of a length is 36cm, then the whole length is $36 \times 4 = 144$ cm

Objective: add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions

Concrete Experience: Use previous knowledge and fraction cards to model the process (including equivalence).
Decide if simplifying or finding equivalence is most effective way of adding or subtracting in each case. Use grids (based on denominators) to model addition/subtraction.

Images:

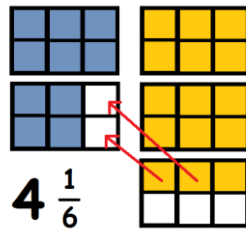
$$\frac{11}{12} - \frac{1}{3} = ?$$



$$- \frac{1}{3}$$

$$1\frac{2}{3} + 2\frac{1}{2}$$

Use denominators to draw grids (in this case 2×3) that represent units.



$$4\frac{1}{6}$$

Language: fraction, part, equal parts, one whole, parts of a whole, number of parts, divide, one half, one third, one quarter, one fifth, one sixth, one tenth, unit fraction, non-unit fraction, numerator, denominator, simplify, equivalence, add, subtract.

Symbols: + -

$$\frac{1}{2} \frac{1}{3} \frac{2}{3} \frac{1}{4} \frac{1}{5} \frac{2}{5} \frac{3}{5} \frac{4}{5} \frac{1}{10} \frac{3}{10} \frac{7}{10} \frac{9}{10} \text{ etc}$$

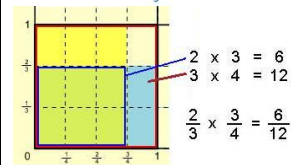
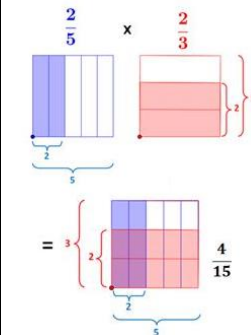
Questions: Of the flags in Jackie's Flag Shop, $\frac{3}{5}$ are green and another $\frac{1}{5}$ are teal. What fraction of the flags are either green or teal?
Sadie's milkshake recipe calls for $\frac{3}{4}$ of a scoop of ice cream and Robbie's recipe calls for $\frac{1}{4}$ of a scoop. How many more scoops of ice cream are used in Sadie's recipe than in Robbie's recipe? How would you model? Can you explain why this is wrong?

Calculating with fractions (decimals, and percentages)

Objective: multiply simple pairs of proper fractions, writing the answer in its simplest form (e.g. $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$)

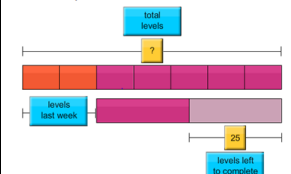
Concrete Experience: Draw grids (or cut up paper grids) to model the process of multiplying fractions. Emphasise the use of 'of' in the place of \times .

Images:



Singapore Bar Method:

Sean is playing Cosmic Blobs on his computer. He finished $\frac{2}{7}$ of the levels last weekend $\frac{1}{2}$ of the remaining levels this week. He had 25 more levels to complete. How many levels does Cosmic Blobs have?



<http://www.mathplayground.com/thinkingblocks.html>

Language: fraction, part, equal parts, one whole, parts of a whole, number of parts, divide, one half, one third, one quarter, one fifth, one sixth, one tenth, unit fraction, non-unit fraction, numerator, denominator, simplify, equivalence, multiply.

Symbols: \times

$$\frac{1}{2} \frac{1}{3} \frac{1}{4} \frac{1}{5} \frac{2}{5} \frac{3}{5} \frac{4}{5} \frac{1}{10} \frac{3}{10} \frac{7}{10} \frac{9}{10} \text{ etc}$$

Questions: What is $a \times b$? How do you know that $c \times d = e$? How can you show me that $f \times g = h$? What is wrong here? Nellie and Brad are collecting clothes for a clothing drive. Brad collected $\frac{3}{4}$ as many clothes as Nellie did. If Nellie collected $\frac{2}{3}$ of a bag of clothes, how many bags of clothes did Brad collect? Last week, Debbie's Fruit Stand sold $\frac{1}{2}$ of a box of melons. Down the road, Angie's Fruit Stand sold $\frac{2}{3}$ as many boxes of melons as Debbie's did. How many boxes of melons did Angie's Fruit Stand sell?

Objective: multiply one-digit numbers with up to two decimal places by one-digit and two-digit whole numbers, such as $0.4 \times 2 = 0.8$, and in practical contexts, such as measures and money

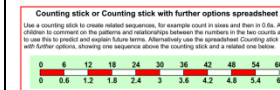
Concrete Experience:

Start with known multiplication facts before relating these to decimal multiplication facts; for example, count on and back in steps of 3 before relating this to counting on and back in steps of 0.3. Or 6, 12, 18, 24, 30, 36, ... \times 0.6, 1. Investigate the relationship between the two sets of numbers. Reinforce the division facts corresponding to multiplication facts; for example: $8 \times 0.7 = 5.6$ $0.7 \times 8 = 5.6$ $5.6 \div 0.7 = 8$ $5.6 \div 8 = 0.7$. Ensure that children meet and can interpret multiplication and division calculations that are written in a variety of different ways, for example: $7 \times 0.8 = 5.6$ $9 = 5.4 \div 0.6$ $0.3 \times 8 = 6 \times 0.4$. Always ask children to check that the size of their answer sounds reasonable. For example, children should recognise that $0.18 \div 2 = 0.9$ cannot be correct. Using a number line to locate numbers such as 0.18 and then thinking about dividing by two could help children to check the reasonableness of an answer.

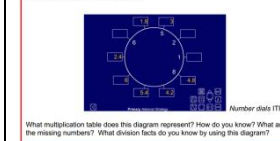
Images: Also see Calculation Policy.

$$42 \div 6 = 7 \quad 0.7 \times 6 = 4.2 \quad 0.07 \times 6 = 0.42$$

$$4.2 \div 6 = 0.7 \quad 7 \times 6 = 42 \quad 0.007 \times 6 = 0.042$$



Use the Number dial ITP to explore multiples of decimal numbers. Explore known multiplication facts, such as multiples of 6, before exploring related facts such as multiples of 0.6.



$$\begin{array}{r} \times \\ 4 \end{array} \begin{array}{r} 1 \\ 4 \end{array} \begin{array}{r} 0.3 \\ 1.2 \end{array} \text{ so } 1.3 \times 4 = 4 + 1.2 = 5.2$$

Language: place value, digit, column, decimal point, tenth, hundredth, thousandth, partition, integer

Symbols: \times \div =

Questions: If $7 \times 6 = 42$, what is 7×0.6 ?

$$0.6 \times \square = 5.4$$

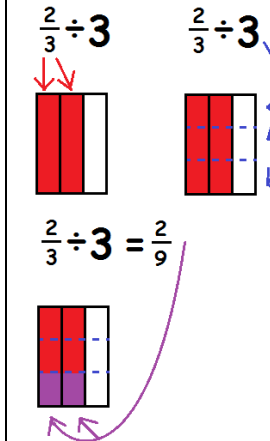
$$5 \square = 0.05$$

Objective: divide proper fractions by whole numbers (e.g. $\frac{1}{2} \div 2 = \frac{1}{4}$)

Concrete Experience:

Draw grids to model the process. Look at the rote method and discuss how and why it works (e.g. $\frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ \rightarrow dividing by 2 is the same as halving).

Images:



Language: fraction, part, equal parts, one whole, parts of a whole, number of parts, divide, one half, one third, one quarter, one fifth, one sixth, one tenth, unit fraction, non-unit fraction, numerator, denominator, simplify, equivalence, divide

Symbols: \div =

$$\frac{1}{2} \frac{1}{3} \frac{1}{4} \frac{1}{5} \frac{2}{5} \frac{3}{5} \frac{4}{5} \frac{1}{10} \frac{3}{10} \frac{7}{10} \frac{9}{10}$$

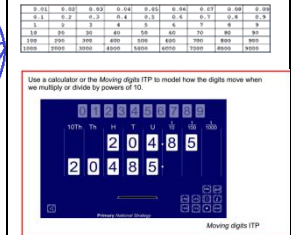
Questions: What is $a \div b$? How can you show me that $f \div g = h$? What is wrong here? I had $\frac{1}{2}$ a cake and wanted to share it between two friends. What fraction of the original cake do they have?

Objective: multiply and divide numbers by 10, 100 and 1000 giving answers up to 3 d.p.

Concrete Experience:

Use place value grids (tens, units, decimal point, tenths, hundredths, thousandths, ...) These could be drawn on paper or made into a wall display for children to hold digit cards against. Create numbers and look at effect of dividing by 10, 100 and 1000. Discuss. Explore the visual pattern of the digits within division.

Images:



Language: digit, decimal, multiply, times, divide, share, scale up, scale down, increase, decrease, factor, tens of thousands, thousands, hundreds, tens, units, ones, tenths, hundredths, thousandths

Symbols: \div

Questions: What is $\dots \div 10 \div 100 \div 1000$? What is $\dots \times 10 \times 100 \times 1000$? How do you know? Can you show me how to multiply/divide by ten/hundred/thousand? What mistake have I made here?

Solve problems

Objective: solve problems which require answers to be rounded to specified degrees of accuracy

Concrete Experience: Use previous knowledge of rounding numbers.
Use RUCSAC (or similar problem solving process) to read, understand (and model), calculate, solve, answer and check. Select relevant models and images used before (selecting most appropriate) and/or Singapore Bar Method (below). Investigate statements about fractions (Always, Sometimes, Never). See Calculation Policy.

Images: All those used before.
Language: All those used before.
Symbols: All those used before.
Questions: Range of questions (see those above), including Always, Sometimes, Never statements etc.

Objective: solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts

Concrete Experience:
Ensure that children meet proportion described in different ways:

- Using everyday language: ten out of 25 children are girls, ten in 25 children are girls.
- In simplified form: two out of every five children are girls, two in every five children are girls.
- As a fraction: $\frac{2}{5}$ of the class are female.
- As a decimal: 0.4 of the class are female.
- As a percentage: 40% of the class are female.

 Ensure that children can use and describe ratios in their simplest form, for example 1:3 is the simplest form of the relationship 3:9.

Images:

Number lines or counting stick

A sequence of equivalent ratios is produced when a given ratio is scaled up by a factor of one, then two, then three... This can be represented on a number line or counting stick. Encourage children to describe the patterns within the sequence.

Ratio and proportion ITP

This program enables you to set the ratio for yellow/pink liquid that is poured into two measuring cylinders. This provides a visual image for the relationship between two quantities in this ratio. A scale factor can be used that scales up the amount of each colour poured in.

Fraction ITP

This program allows you to break a strip into different parts and to colour some of the parts yellow. The program can display the ratio of yellow to green parts.

Line graphs

Graphs can be used to compare related measurements. Where the measurements are in a constant ratio, for example, in conversions between units of measurement or currencies, the graph formed will be a straight-line graph.

Singapore Bar Method:

Dana and Kendra share some erasers in the ratio 3:4. Kendra has 40 erasers. How many erasers do they have altogether?

<http://www.mathplayground.com/thinkingblocks.html>

Language: ratio, for every, to every, equivalent, simplify, problem, pattern, relationship, scale up/down

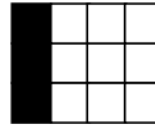
Symbols: % . \times =

Questions: The ratio of girls to boys at the school play is 5:4. There are 44 boys. How many more girls are there than boys? The sum of two numbers is 40. The ratio of the bigger number to the smaller number is 3:2. What is the smaller number? Mrs Ito has 25 students in her class. 2 out of 5 students stayed after school yesterday for play practice. The other students stayed for band practice. How many students stayed for band practice?

Ratio and Proportion

Objective: solve problems involving similar shapes where the scale factor is known or can be found

Concrete Experience:
Rehearse scaling proportions up and down. This technique can be used to solve problems. Provide visual images, for example: one in four tiles is black two in eight tiles is black three in twelve tiles are black



The ratio of a distance

on a map to the Rehearse scaling ratios up/down. This technique can be used to solve problems:

- 5 miles is approximately equal to 8 km (1 kilometre is \approx 0.6 of a mile)
- 10 miles is approximately equal to 16 km
- 15 miles is approximately equal to 24 km

Images:

Number lines and scales
Children need to be able to work out the value of each interval on a number line using the proportion that it represents of a known amount.

Scale drawings, models and scaled maps

The ratio of a length on a drawing, model or map to the equivalent length on the real item is given by the scale.

Singapore Bar Method:

The ratio of the height of a rectangle to its width is 3:2. If the height of the rectangle is 12 centimetres, what is its width?

<http://www.mathplayground.com/thinkingblocks.html>

Language: ratio, scale, up, down, multiply, divide, factors, multiples, quotients, fractions.

Symbols: $\times \div \approx =$

Questions: The ratio of the height of a rectangle to its width is 3:2. If the height of the rectangle is 12 centimetres, what is its width?

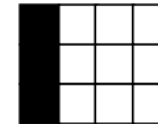
Two numbers are in the ratio 3:2. If the smaller number is 20, what is the bigger number?
For every 4 books that Lily sold, Monica sold 3. Lily sold 16 books last month. How many books did Monica sell? The ratio of girls to boys in Mrs Wang's class is 3:2. If there are 24 boys, how many girls are in the class?

Objective: solve problems involving unequal sharing and grouping using knowledge of fractions and multiples

Concrete Experience:
Provide visual images for ratios then ask children to describe the scenario using the language and notation of ratio, and vice versa.



Each cone has two scoops of chocolate ice cream to every one scoop of strawberry. Ensure that children understand and can use ratios described in different ways:



Using everyday language: there is one black tile to three white tiles; there is one black tile for every three white tiles.

Using a colon (use everyday language first, then the colon form). The ratio of black tiles to white tiles is one to every three. The ratio of black tiles to white tiles is 1:3. The ratio of white tiles to black tiles is 3:1.

Images:

Area ITP or tiles or squared paper
Rows of coloured tiles or squares can be displayed in a given ratio to create a sequence of equivalent ratios.

Ratio and proportion ITP

Use this program to set the ratio for yellow/pink liquid to provide a visual image for the relationship between the two quantities. The yellow and pink liquid can be combined in a single measuring cylinder to explore what proportion of the total mixture is each colour.

Singapore Bar Method:

The sum of two numbers is 40. The ratio of the bigger number to the smaller number is 3:2. What is the smaller number?

<http://www.mathplayground.com/thinkingblocks.html>

Language: problem, pattern, relationship, ratio, proportion, in every, for every, to every, fraction, equivalent, simplify

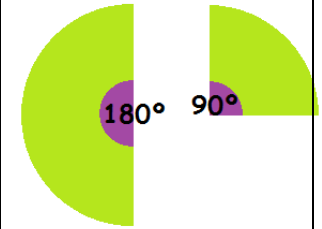
Symbols: $\times \div = /$

Questions: The ratio of the number of flags in Eric's collection to the number of flags in Will's collection is 5:4. Will has 16 flags. How many flags do they have altogether?
The ratio of jazz tunes to country tunes on Kayla's mp3 player is 6:5. She has 3 more jazz tunes than country tunes. How many jazz tunes does Kayla have?

Objective: link percentages or 360° to calculating angles of pie charts

Concrete Experience:
Use a circle to represent a pie chart. Fold and cut it to find 50% ($\frac{1}{2}$ or 0.5), 25% ($\frac{1}{4}$ or 0.25) and so on. At each stage measure the angle with a protractor to ascertain 50% = 180° and so on.

Images:



Comedy	Action	Romance	Drama	SciFi	TOTA
4	5	6	1	4	20
$\frac{4}{20} = 20\%$	$\frac{5}{20} = 25\%$	$\frac{6}{20} = 30\%$	$\frac{1}{20} = 5\%$	$\frac{4}{20} = 20\%$	100%
$\frac{4}{20} \times 360^\circ = 72^\circ$	$\frac{5}{20} \times 360^\circ = 90^\circ$	$\frac{6}{20} \times 360^\circ = 108^\circ$	$\frac{1}{20} \times 360^\circ = 18^\circ$	$\frac{4}{20} \times 360^\circ = 72^\circ$	360°

Language: angles, percentages, fraction,

Symbols: $=$ ° % \times / .

Questions: How many degrees are there in a whole turn or a whole pie chart? What is 50% (or $\frac{1}{2}$) of the angles in a pie chart? What is 25% (or $\frac{1}{4}$) of a pie chart? How can you show me? Can you fold this circle to show me different fractions? What percentage is each fraction? How many angles are in each fraction/percentage?