

Bellfield Primary School Progression in Calculations

Addition

Addition Foundation stage

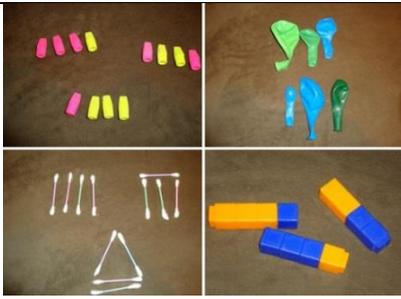
<p>22-36 months:</p> <p>Knows that a group of things changes in quantity when something is added</p>	<p>30 - 50 months:</p> <p>Separates a group of three or four objects in different ways, beginning to recognise that the total is still the same.</p>	<p>40-60+ months:</p> <p>Finds the total number of items in two groups by counting all of them.</p> <p>Says the number that is one more than a given number.</p>	<p>Early Learning Goal:</p> <p>In practical activities and discussion, beginning to use the vocabulary involved in adding</p> <p>Add two single digits together and find the answer</p> <p>Notes: Introduction to number sentences. Objects used, move objects when counted. Move to counting on and counting starting at different numbers.</p>
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Objective and Strategies	Concrete	Pictorial	Abstract
<p>22-36 months:</p> <p>Knows that a group of things changes in quantity when something is added</p>	 		

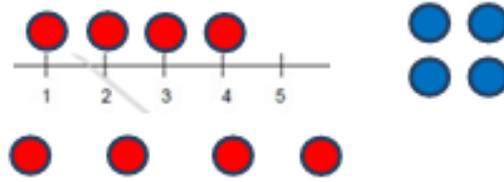
30 - 50

months:

Separates a group of three or four objects in different ways, beginning to recognise that the total is still the same.



Matching objects to numerals on number lines support conservation of number.



40-60+
months:

Finds the total number of items in two groups by counting all of them.



In practical activities and discussion, beginning to use the vocabulary involved in adding



Find the total number of balls shown here.



5 balls



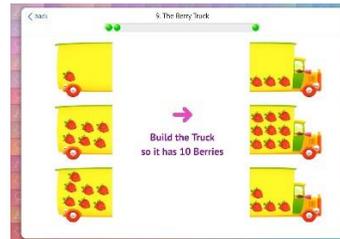
and 3 balls



total

Action - vocab - symbols

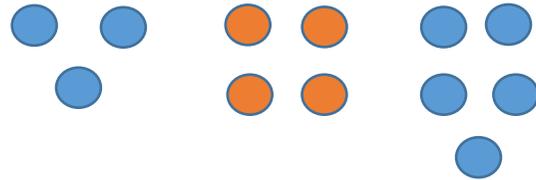
Do it - say it -record it



Says the number that is one more than a given number



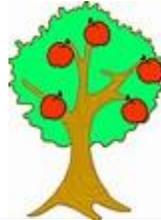
Ask children to gather objects showing one more or less than a given number



Given an amount of object children to represent the amount that is one more or less.

Complete images to represent the amount one more/less.

e.g The apple tree in Miss Howard's garden has 4 apples. Mrs Scowen's apple tree has one more apple than mine. Can you draw the apples on Mrs Scowen's tree?

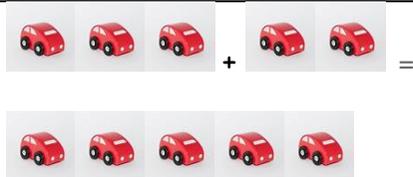


Children to show the amount using the numeral.



Early learning goal

Using quantities and objects, they add two single-digit numbers and count on to find the answer.

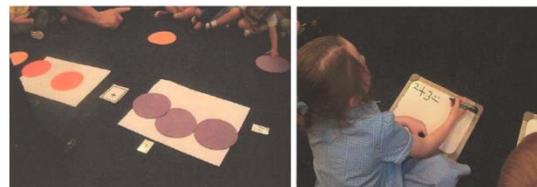
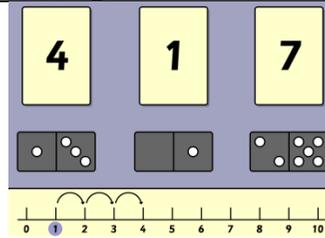


At this stage children to rehearse number sentences orally



6 add 3 equals 9

6 + 3 = 9



Children add amounts by placing the largest number in their heads and counting on.

Children are introduced to the abstract number sentence but pictorial or concrete objects to remain alongside at this stage

- 1+1 =
- 2+2 =
- 3+3 =
- 4+4 =
- 5+5 =
- 2+1 =
- 1+2 =



$$5 + 3 =$$

Addition Year 1

Add with numbers up to 20

Develop ways of recording using pictorial representations alongside practical equipment.

Use numbered number lines to add, by counting on in ones. Encourage children to start with the larger number and count on.

Key Vocabulary

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line

Key skills for addition at Y1:

Read and write numbers to 100 in numerals, incl. 1–20 in words
 Recall bonds to 10 and 20, and addition facts within 20
 Count to and across 100
 Count in multiples of 1, 2, 5 and 10
 Solve simple 1-step problems involving addition, using objects, number lines and pictorial representations

Links

Video clips: [Using a range of equipment and strategies to reinforce addition statements](#)

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Mental

Number bonds to 20
 Add and subtract 10 to a 2 digit number
 Add and subtract 1 to a 2 digit number
 Adding within 10
 Know doubles to 10

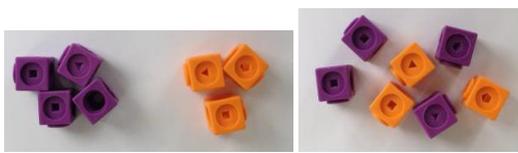
**Objective
and
Strategies**

Concrete

Pictorial

Abstract

Combining two parts to make a

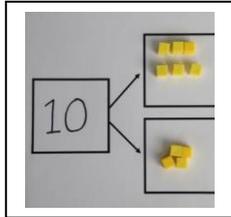


Use cubes to add two numbers together as a group or in a bar.

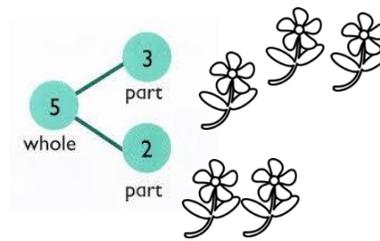
whole model



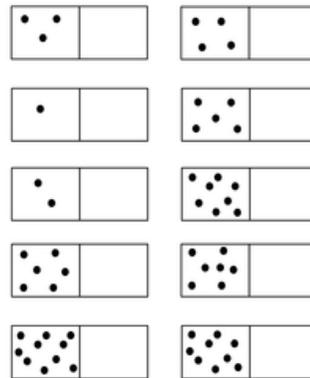
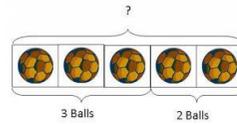
(Number bonds)



Recognise that addition can be done in any order (**switcher rule**)

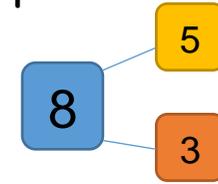


Use pictures to add two numbers together as a group or in a bar.

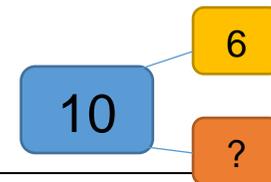
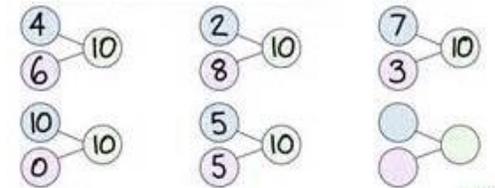


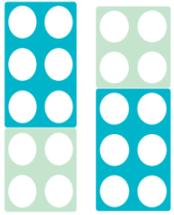
$$4 + 3 = 7$$

$$10 = 6 + 4$$

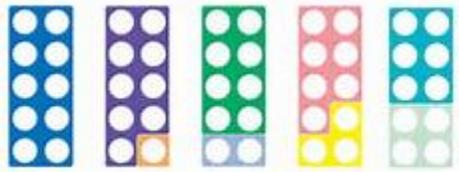


Use the part-part whole diagram as shown above to move into the





$4+6 \quad 6+4$



$10 = \quad 1+9 \quad 2+8 \quad 3+7 \quad 4+6$

Starting at the bigger number and counting on



Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.

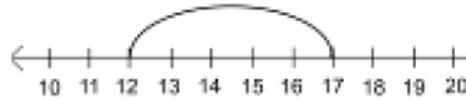
5

There are 5 knights in the castle.

+

3 more knights join them.

$12 + 5 =$
 17



Start at the larger number on the number line and count on in ones or in one jump to find the answer.

$2 \quad \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} =$

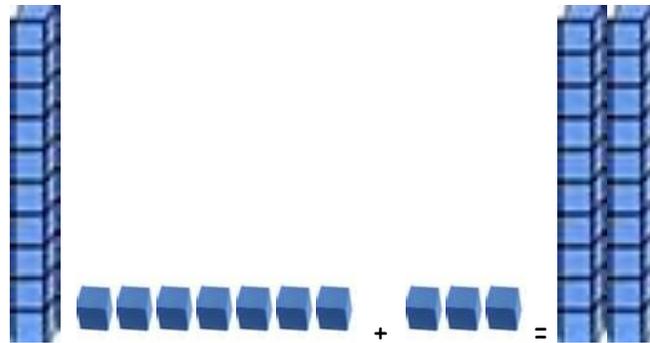
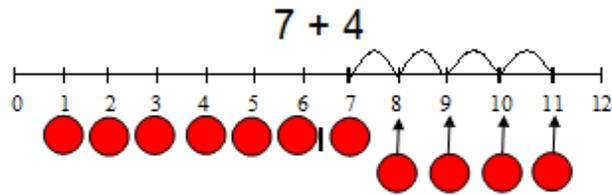
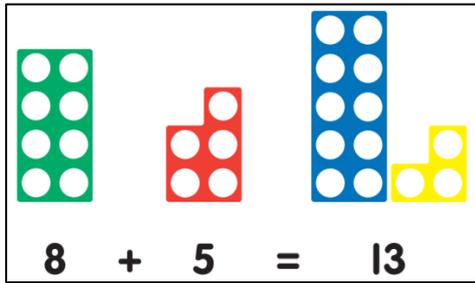
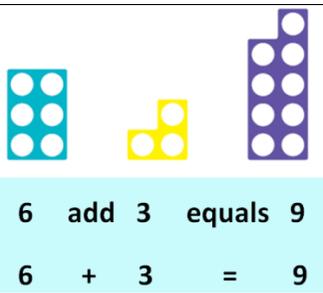
Representing amount is various ways where the first number is given and images for the amount to be added on.

+

9 + **3** =

$5 + 12 = 17$

Place the larger number in your head and count on the smaller number to find your answer.

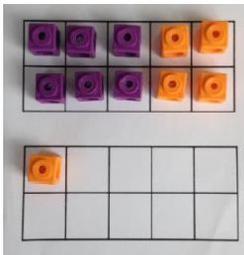


Regrouping to make 10.

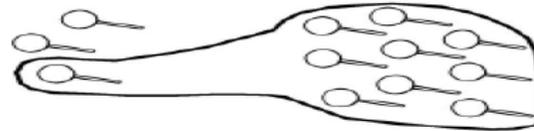
$9 + 3 = 12$



$6 + 5 = 11$

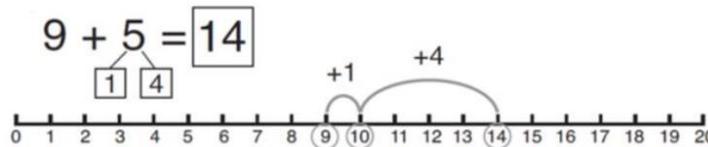


Tens Frames
 Start with the bigger number and use the smaller number to make 10.



$3 + 9 =$

Use pictures or a number line. Regroup or partition the smaller number to make 10.



$7 + 4 = 11$

If I am at seven, how many more do I need to make 10. How many more do I add on now?

Addition Year 2
Add with 2-digit numbers

Developing mental fluency with addition and place value involving 2-digit numbers, then establish more formal methods

Key Vocabulary

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary

Key skills for addition at Y2:

Add a 2-digit number and ones (e.g. 27 + 6)
 Add a 2-digit number and tens (e.g. 23 + 40)
 Add pairs of 2-digit numbers (e.g. 35 + 47)
 Add three single-digit numbers (e.g. 5 + 9 + 7)
 Show that adding can be done in any order (the commutative law).
 Recall bonds to 20 and bonds of tens to 100 (30 + 70 etc.)
 Count in steps of 2, 3 and 5 and count in tens from any number.
 Understand the place value of 2-digit numbers (tens and ones)
 Compare and order numbers to 100 using < > and = signs.
 Read and write numbers to at least 100 in numerals and words.
 Solve problems with addition, using concrete objects, pictorial representations, involving numbers, quantities and measures, and applying mental and written methods.

Links

Video clips: <http://www.youtube.com/watch?v=HGk8F6rRpPg>
<http://www.topmarks.co.uk/Flash.aspx?f=AddExpandv2>
 Really nice Gordon's expanded addition interactive white board demo

Mental

Recall addition facts to 20 fluently

Complements of numbers to 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20.

Derive and use related facts to 100

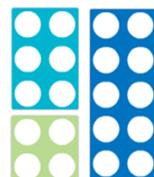
$3 + 2 = 5$ $30 + 20 = 50$

Commutative rule (Swopys rule)



$6 + 4$ $4 + 6$

Addition and subtraction as inverses



$4 + 6 = 10$

$6 + 4 = 10$

$10 - 6 = 4$

$10 - 4 = 6$

To recall number bonds with totals to 20

Know pairs of multiples of 10 with totals up to 100, e.g. 30 + 70, or 60 + \square = 100

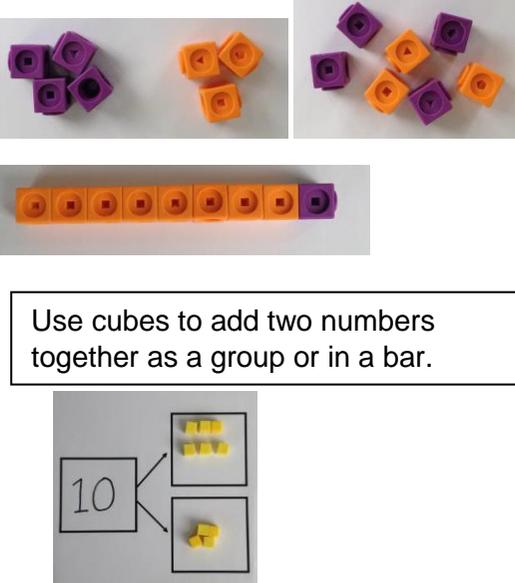
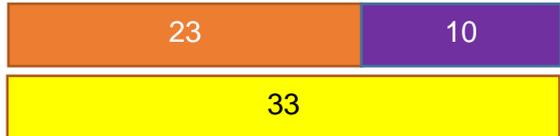
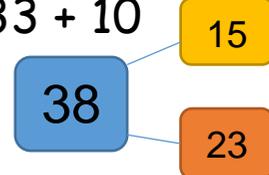
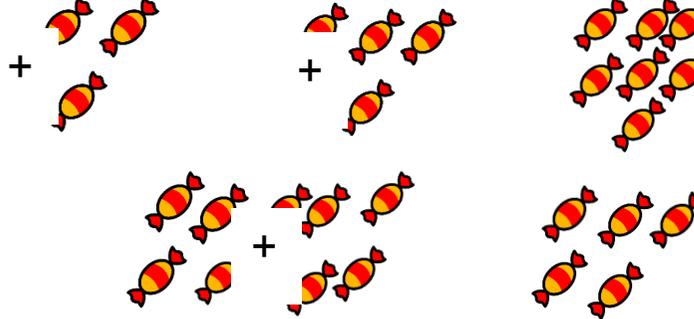
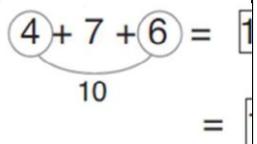
To know what must be added to any two-digit number to make the next multiple of 10 To use knowledge of pairs making 10 when calculating mentally

To add a multiple of 10 and adjust by 1

Add and subtract multiples of 10 to any give 2-digit number

Say 10 more/less than any number to 100

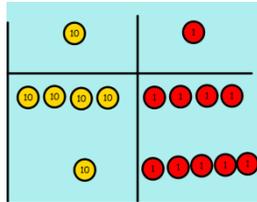
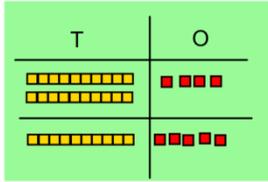
Add two or three single digit numbers

Objective and Strategies	Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole: part-whole model</p>	 <p>Use cubes to add two numbers together as a group or in a bar.</p>	<p>Use pictures to add two numbers together as a group or in a bar.</p> 	<p>$15 + 23 = 38$</p> <p>$43 = 33 + 10$</p>  <p>Use the part-part whole diagram as shown above to move into the abstract.</p>
<p>Adding three single digits</p>	<p>$4 + 7 + 6 = 17$ Put 4 and 6 together to make 10. Add on 7.</p>  <p>Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.</p>	 <p>Add together three groups of objects. Draw a picture to recombine the groups to make 10</p>	 <p>Combine the two numbers that make 10 and then add on the remainder.</p>

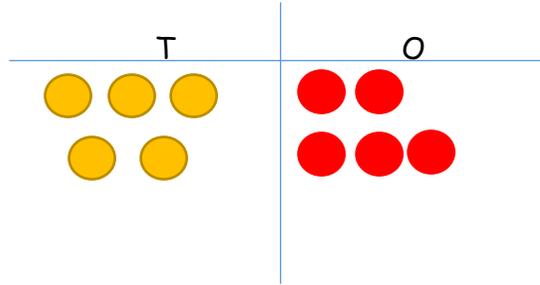
Column
method- no
regrouping
Set out in
columns

$$24 + 15 =$$

Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.



After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.



2	0	+	3		
+	3	0	+	4	

5	0	+	7		
			= <u>57</u>		

Expanded method for higher ability
if/when ready:

$$\begin{array}{r}
 47 \\
 + 76 \\
 \hline
 13 \\
 110 \\
 \hline
 123
 \end{array}$$

Addition Year 3
Add numbers with up to three digits

Addition Year 4
Add numbers up to four digits

Key
Vocabulary

Y
 add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, =, carry', expanded, compact

Key skills for addition at
Y3:

Estimate answers to calculations, using inverse to check answers.
 Solve problems, including missing number problems, using number facts, place value, and more complex addition.
 Recognise place value of each digit in 3-digit numbers (hundreds, tens, ones.)
 Continue to practise a wide range of mental addition strategies, ie. number bonds, adding the nearest multiple of 10, 100, 100 and adjusting, using near doubles, partitioning and recombining.
 Add 2-digit numbers mentally, incl. those exceeding 100.
 Add a three-digit number and ones mentally (175 + 8)
 Add a three-digit number and tens mentally (249 + 50)
 Add a three-digit number and hundreds mentally (381 + 400)

Links

Video Link:
Demonstration of
expanded 3 -digit
column addition

<http://www.enchantedlearning.com/math/add/>

Really good worksheets including column addition and good addition problems

<http://www.topmarks.co.uk/Flash.aspx?f=AddExpanded2>

Really nice Gordon's expanded addition interactive white board demo

<http://www.mathsisfun.com/numbers/addition-column.html>

Interactive examples of column addition (spot deliberate tens mistake)

Key
Vocabulary

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, „carry“, expanded, compact, thousands, hundreds, digits, inverse

Key skills for addition at
Y4:

Select most appropriate method: mental, jottings or written and explain why.
 Recognise the place value of each digit in a four-digit number.
 Round any number to the nearest 10, 100 or 1000.
 Estimate and use inverse operations to check answers.
 Solve 2-step problems in context, deciding which operations and methods to use and why.
 Find 1000 more or less than a given number.
 Continue to practise a wide range of mental addition strategies, ie. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombining.
 Add numbers with up to 4 digits using the formal written method of column addition
 Solve 2-step problems in contexts, deciding which operations and methods to use and why.
 Estimate and use inverse operations to check answers to a calculation

Mental

Counting for calculating:

- 3D + 1D
- 3D + multiple of ten
- 3D + multiple of hundred



Add 2-digit numbers mentally, incl. those exceeding 100.

Add a three-digit number and ones mentally ($175 + 8$)

Add a three-digit number and tens mentally ($249 + 50$)

Add a three-digit number and hundreds mentally ($381 + 400$)

Add and subtract multiples of 10

Use knowledge of number bonds to find bonds with a total of 100

Addition doubles for multiples of 10 to 100, e.g. $90 + 90$

Add or subtract a two-digit number to or from a multiple of 10, e.g. $50 + 38$, $90 - 27$

To add near doubles, e.g. $18 + 16$, $60 + 70$

Add or subtract 10 or 20 and adjust

Find 10 or 100 more/less than a given number.

Mental

Continue to practise a wide range of mental addition strategies, ie. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombining.

I can find the nearest multiple of

10

100

1000

Find 1000 more/less than a given number.

To find the sum and differences of pairs of multiples of 10, 100 or 1000

Add or subtract any pair of two-digit numbers, including crossing the tens and 100 boundary, e.g. $47 + 58$, $91 - 35$

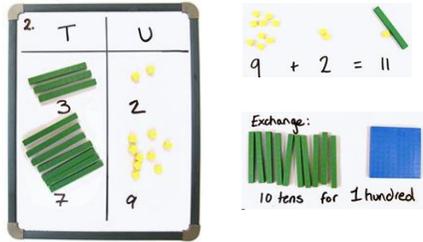
Add or subtract two-digit or three-digit multiples of 10, e.g. $120 - 40$, $140 + 150$, $370 - 180$

Add tens and ones separately, then recombine

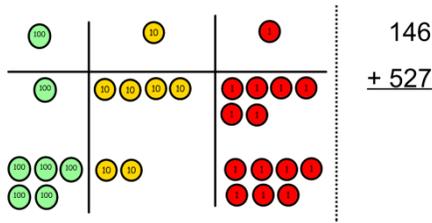
Add or subtract a multiple of 10 and adjust, e.g. $56 + 29 = 56 + 30 - 1$, or $86 - 38 = 86 - 40 + 2$

Objective and Strategies

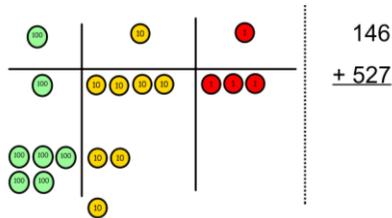
Column method-regrouping



Make both numbers on a place value grid.



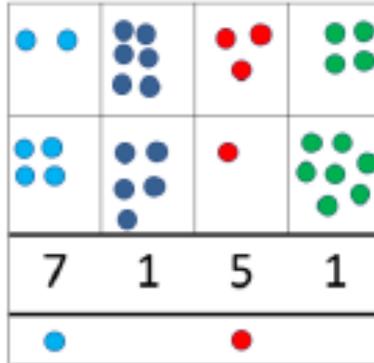
Add up the units and exchange 10 ones for one 10.



Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

Pictorial

Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.



Abstract

	H	T	U
		3	2
+		7	9
		1	1
	1	0	0
	1	1	1

Start by partitioning the numbers before moving on to clearly show the exchange

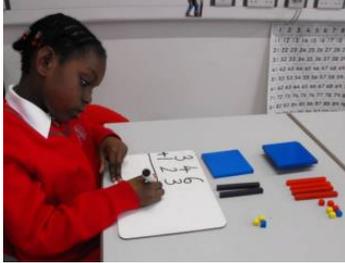
$$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$$

	H	T	U
		3	2
+		7 ₁	9
	1	1	1

$$\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$$

Year 4 As the children move on, introduce decimals with the same number of decimal places and different.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.



Concrete apparatus and visual imagery is essential. Some children may need this throughout KS2.

Year 4: As children move on to decimals, money and decimal place value counters can be used to support learning.

$$\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \\ 11 \end{array}$$

$$\begin{array}{r} \pounds 23.59 \\ + \pounds 7.55 \\ \hline \pounds 31.14 \\ 111 \end{array}$$

Money can be used here.

$$\begin{array}{r} 23.361 \\ 9.080 \\ 59.770 \\ + 1.300 \\ \hline 93.511 \\ 212 \end{array}$$

Year 5

Add numbers with up to five digits including money, measures and decimals with different numbers of decimal places

Key Vocabulary

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, „carry“, expanded, compact, vertical, thousands, hundreds, digits, inverse & decimal places, decimal point, tenths, hundredths, thousandths

Key skills for addition at Y5:

Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies ie. add the nearest multiple of 10, 100, 100 and adjust; use near doubles, inverse, partitioning and re-combining; using number bonds. Use rounding to check answers and accuracy. Solve multi-step problems in contexts, deciding which operations and methods to use and why. Read, write, order and compare numbers to at least 1 million and determine the value of each digit. Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000. Add numbers with more than 4 digits using formal written method of columnar addition.

Year 6

Add several numbers of increasing complexity

Key Vocabulary

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, „carry“, expanded, compact, vertical, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths

Key skills for addition at Y6

Solve multi-step problems in context, deciding which operations and methods to use and why. Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy. Read, write, order and compare numbers up to 10 million and determine the value of each digit. Round any whole number to a required degree of accuracy. Pupils understand how to add mentally with larger numbers and calculations of increasing complexity.

Mental

Use place value and number facts to add two or more friendly numbers including money and decimals (e.g. $3+4+8+6+7$, $0.6+0.4+0.7$)
 Add and subtract decimal numbers which are near multiples of 1 or 10 including money (e.g. $£6.34-1.99$ or $£34.59-£19.95$)
 Add to the next 10 from a decimal number
 To know what must be added to any four-digit number to make the next multiple of 10, 100 or 1000, e.g. $4087 + \square = 5000$
 To know what must be added to a decimal with units and tenths to make the next whole number, e.g. $7.2 + \square = 8$
 Add or subtract a pair or 2 digit numbers or 3 digit multiples of 10
 Add or subtract a near multiple of 10 or 100 to any two-digit or three-digit number, e.g. $235 + 198$
 Add or subtract a multiple of 10 or 100 and adjust find 50%, 25% or 10% of whole numbers or quantities, e.g. 25% of 20 kg, 10% of £80

<http://www.enchantedlearning.com/math/add/>

Really good worksheets including column addition and good addition problems

Mental

Add positive number to negative numbers (e.g. calculate a rise in temp)
 Add two 1-place decimal numbers or two 2-place decimal numbers less than 1 ($4.5 + 6.5$ or $0.74 + 0.33$)
 Derive quickly and without difficulty, number bonds to 1000
 Use number bonds to 1 and 10 to perform mental subtraction of any pair of one decimal place numbers
 Addition and subtraction facts for multiples of 10 to 1000 and decimal numbers with one decimal place

Objective and Strategies

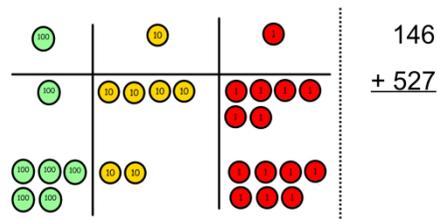
Concrete

Pictorial

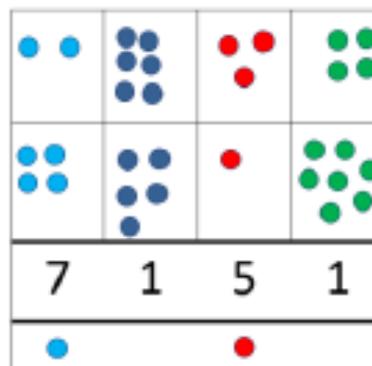
Abstract

Column method- regrouping

Make both numbers on a place value grid.



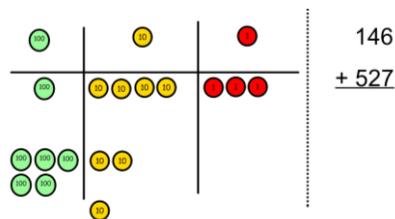
Children can draw a pictorial representation of the columns and place value counters to further



	TH	H	T	U	.	th	hth
		3	5	4	.	5	6
	1	1	0	3	.	3	4
+			2	0 ₁	.	1 ₁	2
	1	4	7	8	.	0	2

£	2	3	.	5	9
+	£	7	.	5	5
£	3	1	.	1	4

Add up the units and exchange 10 ones for one 10.



Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

support their learning and understanding.

The decimal point should be aligned in the same way as the other place value columns, and must be in the same column in the answer.

Numbers should exceed 4 digits.

Pupils should be able to add more than two values, carefully aligning place value columns.

$$\begin{array}{r}
 4.2 \quad \rightarrow \quad 4.200 \\
 0.34 \quad \rightarrow \quad 0.340 \\
 5.871 \quad \rightarrow \quad 5.871 \\
 + 18 \quad \rightarrow \quad 18.000 \\
 \hline
 \mathbf{28.411}
 \end{array}$$

Adding several numbers with different numbers of decimal places (including money and measures):

$$\begin{array}{r}
 23.361 \\
 9.080 \\
 59.770 \\
 + 1.300 \\
 \hline
 93.511 \\
 \substack{212}
 \end{array}$$

- Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.

- Zeros could be added into any empty decimal places, to show there is no value to add.

Subtraction

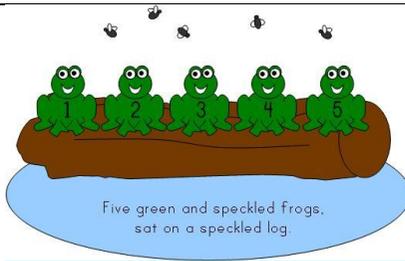
Subtraction Foundation stage

<p>22-36 months:</p> <p>Knows that a group of things changes in quantity when something is taken away</p>	<p>30 - 50 months:</p> <p>Shows an interest in simple subtraction number problems</p>	<p>40-60+ months:</p> <p>In practical activities and discussions, begins to use vocabulary involving subtraction</p>	<p>Early Learning Goal:</p> <p>Says the number that is one less than a given number to 20.</p> <p>In practical activities and discussion, beginning to use the vocabulary involved in subtraction</p> <p>Subtract by counting back</p> <p>Notes: Introduction to number sentences. Objects used, move objects when counted. Move to counting backwards and counting backwards from at different starting numbers.</p>
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Objective and Strategies	Concrete	Pictorial	Abstract
<p>22-36 months:</p> <p>Knows that a group of things changes in quantity when something is taken away</p>			

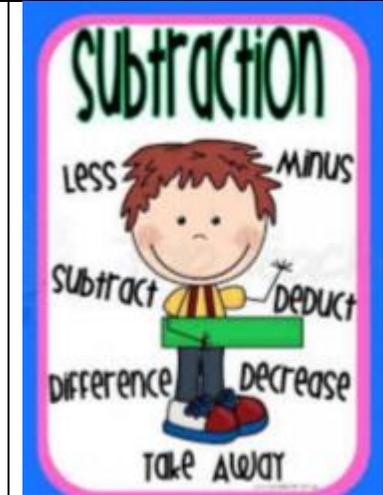
30 - 50 months:

Shows an interest in simple subtraction number problems



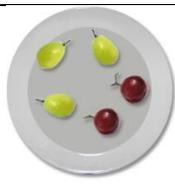
40-60+ months:

In practical activities and discussions, begins to use vocabulary involving subtraction



Early Learning Goals

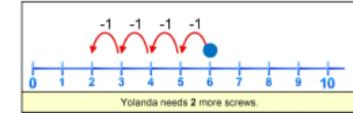
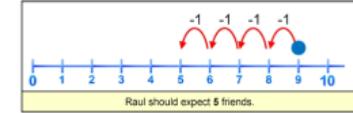
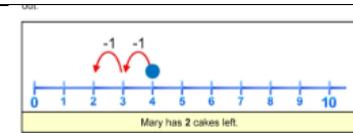
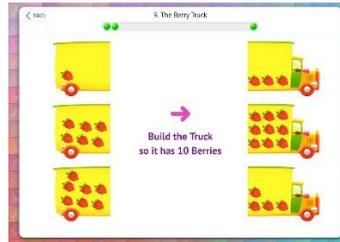
Using quantities and objects, they subtract two single-digit numbers and count back to find the answer.



In practical activities and discussion, beginning to use the vocabulary involved in subtraction



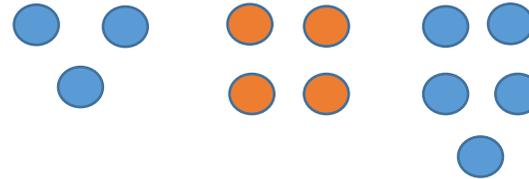
Action - vocab - symbols
Do it - say it - record it



Says the number that is one more than a given number



Ask children to gather objects showing one more or less than a given number



Given an amount of object children to represent the amount that is one more or less.

Complete images to represent the amount one more/less.

e.g The apple tree in Miss Howard's garden has 4 apples. Mrs Scowen's apple tree has one more apple than mine. Can you draw the apples on Mrs Scowen's tree?



Children to show the amount using the numeral.



Subtraction Year 1
Subtract from numbers up to 20

Key Vocabulary

equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_?

Key skills for subtraction at **Y1:**

Given a number, say **one more or one less**.
Count to and over 100, **forward and back**, from any number.
Represent and use **subtraction facts to 20 and within 20**.
Subtract with **one-digit and two-digit** numbers to 20, including zero.
Solve one-step problems that involve addition and subtraction, using concrete objects (ie bead string, objects, cubes) and pictures, and missing number problems.
Read and write numbers from 0 to 20 in numerals and words.

Links

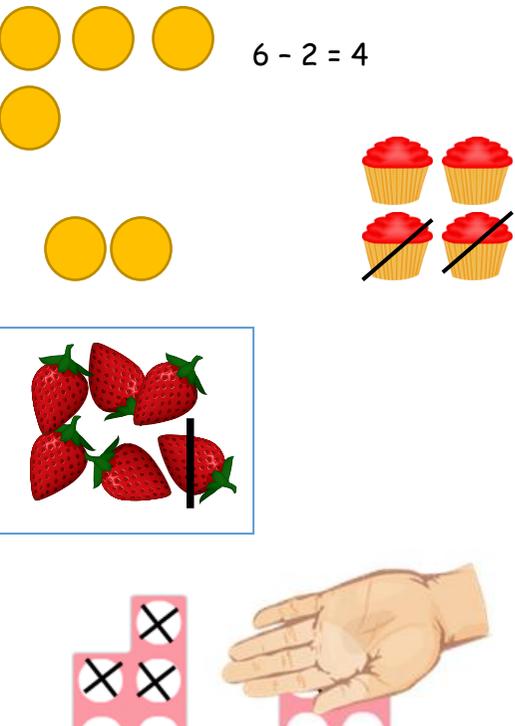
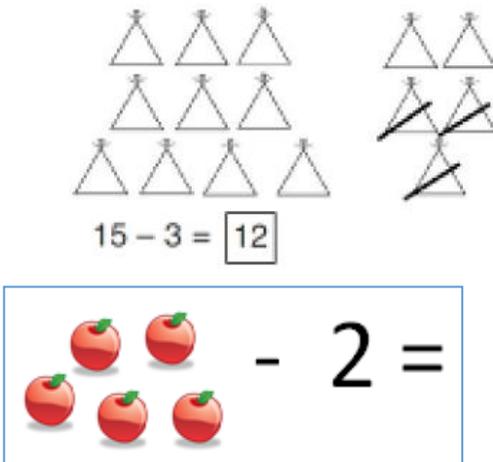
Video Clip - Addition and Subtraction Key Stage 1

<https://www.ncetm.org.uk/resources/40533>

Mental

Number bonds to 20
Subtract any 1 digit number from any 2 digit number
Add and subtract 10 to a 2 digit number
Add and subtract 1 to a 2 digit number
Subtract within 10

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Objective and Strategies	Concrete	Pictorial	Abstract
<p>Taking away ones</p>	<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p>  <p>6 - 2 = 4</p>	<p>Cross out drawn objects to show what has been taken away.</p>  <p>15 - 3 = 12</p> <p>5 - 2 =</p>	<p>18 - 3 = 15</p> <p>8 - 2 = 6</p>

Counting back

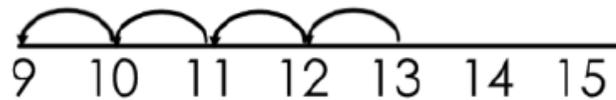
Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.

$13 - 4$

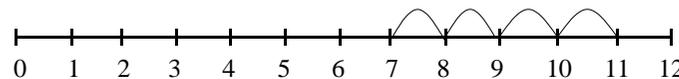


Use counters and move them away from the group as you take them away counting backwards as you go.

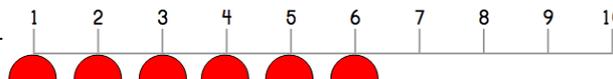
Count back on a number line or number track



Start at the bigger number and count back the smaller number showing the jumps on the number line.

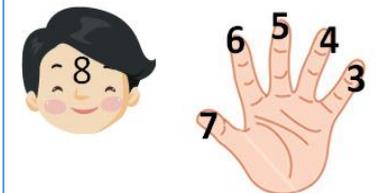


$11 - 4 =$



Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

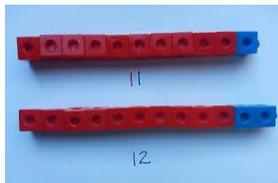
$8 - 5 =$



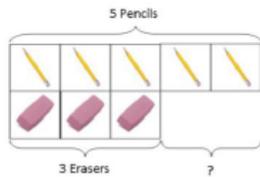


Find the difference

Compare amounts and objects to find the difference.

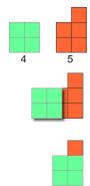


Use cubes to build towers or make bars to find the difference

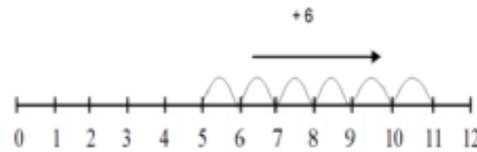


Use basic bar models with items to find the difference

difference



With Numicon place the tiles one on top of the other so that the 'difference' is clearly visible...

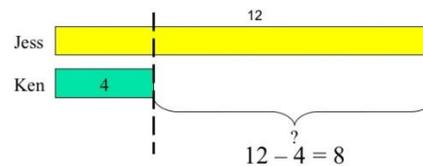


Count on to find the difference.

Draw bars to find the difference between 2 numbers.

Comparison Model (to find the difference)

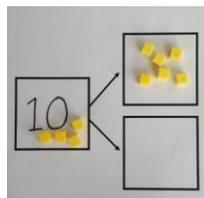
Jess had 12 beads and Ken had 4.
How many more beads had Jess than Ken?



What do you think is the common mistake made by many students?

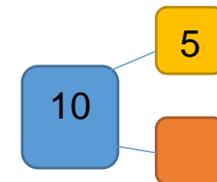
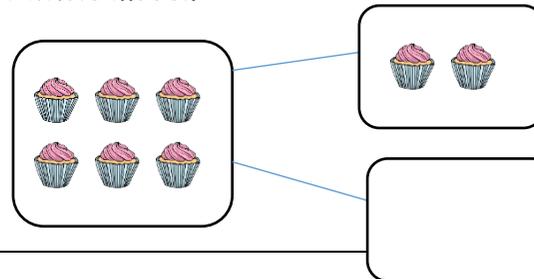
Hannah has 13 sandwiches, Helen has 8 sandwiches. Find the difference between the number of sandwiches.

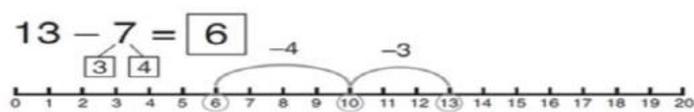
Part Part Whole Model



Link to addition- use the part whole model to help explain the inverse between addition and subtraction.

Use a pictorial representation of objects to show the part part whole model.



	<p>If 10 is the whole and 6 is one of the parts. What is the other part?</p> <p style="text-align: center;">$10 - 6 =$</p>		<p>Move to using numbers within the part whole model.</p>
<p>Make 10</p>	<p>$14 - 9 =$</p>  <p>Make 14 on the ten frame. Take away the four first to make 10 and then take away one more so you have taken away 5. You are left with the answer of 9.</p>	 <p>$13 - 7 = 6$</p> <p>Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.</p>	<p>$16 - 8 =$</p> <p>How many do we take off to reach the next 10?</p> <p>How many do we have left to take off?</p>

Subtraction Year 2

Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: two digit number and ones, two digit numbers tens, two two digit numbers; adding three one digit numbers

<p><u>Key Vocabulary</u></p> <p>equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, units</p>	<p><u>Key skills for subtraction at Y2:</u></p> <p>Recognise the place value of each digit in a two-digit number. Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100. Subtract using concrete objects, pictorial representations, 100 squares and mentally, including: a two- digit number and ones, a two-digit number and tens, and two two-digit numbers. Show that subtraction of one number from another cannot be done in any order. Recognise and use inverse relationship between addition and subtraction, using this to check calculations and missing number</p>	<p><u>Links</u></p> <p>Video Clip - Addition and Subtraction Key Stage 1</p> <p>https://www.ncetm.org.uk/resources/40533</p>
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problems.
Solve simple addition and subtraction problems including measures, using concrete objects, pictorial representation, and also applying their increasing knowledge of mental and written methods.
Read and write numbers to at least 100 in numerals and in words.

Mental

Recall subtraction facts to 20 fluently
Compliments of numbers to 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20.

Derive and use related facts to 100
 $5 - 2 = 3$ $50 + 20 = 30$

Addition and subtraction as inverses



$4 + 6 = 10$
 $6 + 4 = 10$
 $10 - 6 = 4$
 $10 - 4 = 6$

To subtract a multiple of 10 and adjust by 1
Subtract any pair of 2-digit numbers by counting back in tens and ones or by counting up to find the difference
Add and subtract multiples of 10 to any give 2-digit number
Say 10 more/less than any number to 100

Objective and Strategies

Concrete

Pictorial

Abstract

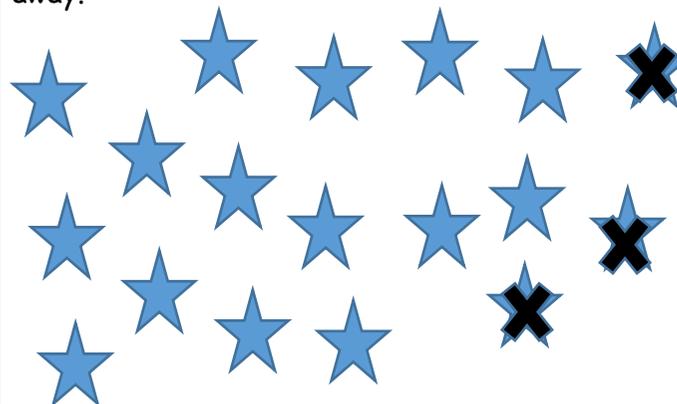
Taking away ones from a two digit number

Use physical objects, counters, cubes etc to show how objects can be taken away.
*move the objects away



$48 - 6 = 42$

Cross out drawn objects to show what has been taken away.



$38 - 6 = 32$

$78 - 9 = 69$



$$20 - 3 = 17$$

Think:
I have 3 tens and 4 ones. I want to take away 9 ones.

Workmat	
Tens	Ones
3 tens	4 ones

Think:
I need more ones. I will regroup 1 ten as 10 ones.

Workmat	
Tens	Ones
2 tens	14 ones

Think:
I now have 2 tens and 14 ones so I can take away 9 ones, leaving 2 tens and 5 ones.

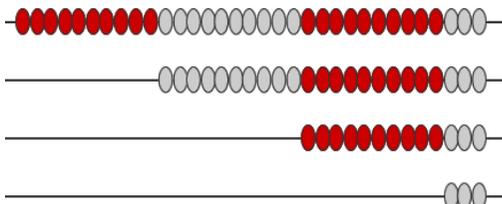
Workmat	
Tens	Ones
2 tens	5 ones

$$2 \text{ tens } 5 \text{ ones} = 25$$

$$18 - 3 = 15$$

Counting back

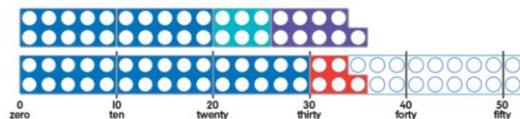
Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in tens.



$$33 - 10 = 23$$

$$23 - 10 = 13$$

$$13 - 10 = 3$$



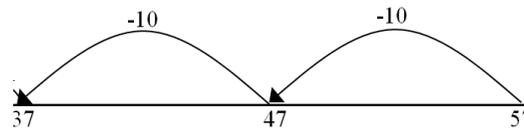
$$35 - 9 = 26$$

Count back on a number line or number track

Start at the bigger number and count back the smaller number showing the jumps on the number line.

$$57 - 10 = 47$$

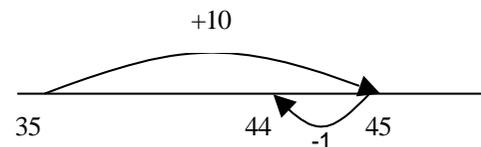
$$47 - 10 = 37$$



Jottings to aid mental calculation

Subtracting 9 or 11 (subtract 10 and adjust by 1)

$$35 + 9 = 44$$

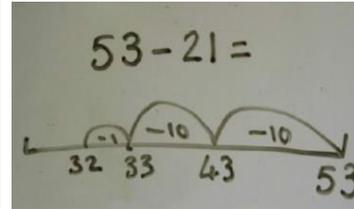
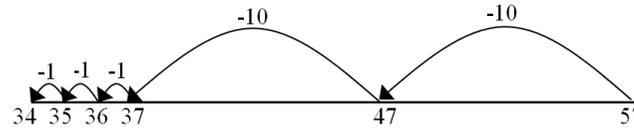


Put 43 in your head, count back 4. What number are you at?

Put 65 in your head and count back 30 (3 tens) what number do you get?

$$85 - 20 =$$

This can progress all the way to counting back using two 2 digit numbers. $57 - 23 = 34$
 Partition the second number and subtract it in tens and units, as below



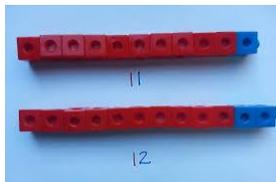
Move towards more efficient jumps back, as below:
 $47 - 23 = 24$



Find the difference

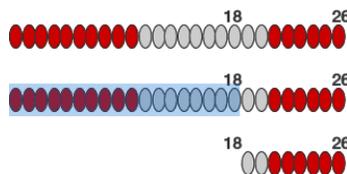
Counting forwards to find the difference
 Mental strategy - subtract numbers close together by counting on:

Compare amounts and objects to find the difference.



Use cubes to build towers or make bars to find the difference

Use bead strings to find the difference

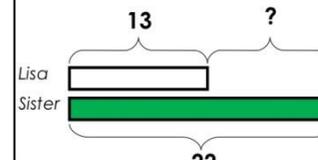
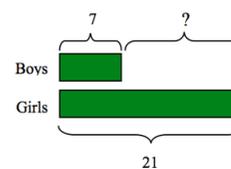


Count on to find the difference on a number line

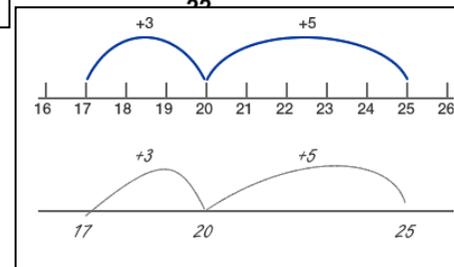
Draw bars to find the

Comparison Bar Models

Lisa is 13 years old. Her sister is 22 years old.
 Find the difference in age between them.



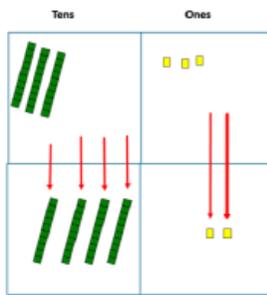
difference between 2 numbers.



Hannah has 13 sandwiches, Helen has 8 sandwiches. Find the difference between the numbers of sandwiches.

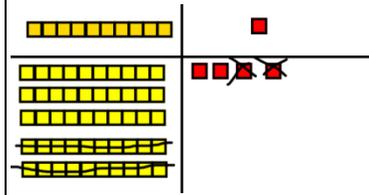
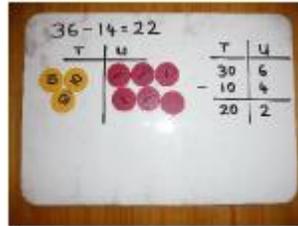
$$38 - 22 = 16$$

Column method without regrouping
Two digit - two digit



Use Base 10 to make the bigger number then take the smaller number away.

Show how you partition numbers to subtract. Again make the larger number first.



help to show working.

Calculations

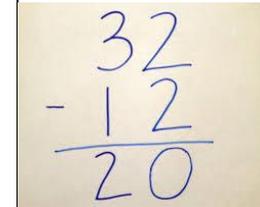
$$\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$$

Draw the Base 10 or place value counters alongside the written calculation to

$$47 - 24 = 23$$

$$\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$$

This will lead to a clear written column subtraction



	T	U
	4	7
-	2	1

Subtraction Year 3

Subtract numbers with up to three digits

Subtraction Year 4

Subtract numbers up to four digits

Key Vocabulary

equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, units **exchange, decrease, hundreds, value, digit**

Mental

Counting for calculating:

- 3D - 1D
- 3D - multiple of ten
- 3D - multiple of hundred



Add and subtract multiples of 10
Use knowledge of number bonds to find bonds with a total of 100
Addition doubles for multiples of 10 to 100, e.g. $90 + 90$
Add or subtract a two-digit number to or from a multiple of 10
To add near doubles, e.g. $18 + 16$, $60 + 70$
Add or subtract 10 or 20 and adjust
Find 10 or 100 more/less than a given number.

Key skills for

subtraction at Y3:

- Subtract mentally a: 3-digit number and ones, 3-digit number and tens, 3-digit number and hundreds .
- Estimate answers and use inverse operations to check.
- Solve problems, including missing number problems.
- Find 10 or 100 more or less than a given number.
- Recognise the place value of each digit in a 3-digit number .
- Counting up differences as a mental strategy when numbers are close together or near multiples of 10 (see examples above)
- Read and write numbers up to 1000 in numerals and words.
- Practise mental subtraction strategies, such as subtracting near multiples of 10 and adjusting (e.g. subtracting 19 or 21), and select most appropriate methods to subtract, explaining why.

Links

Video Link:

Video clips: 1—
[Subtraction—teaching children to consider the most appropriate methods before calculating](#)

Video Clip 2—
[Introducing partitioned column subtraction method, from practical to written](#)

Video: 3—
[Subtraction—teaching children to consider the most appropriate methods before calculating](#)

Key Vocabulary

equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, **inverse**

Mental

Find 1000 more/less than a given number.

To find the sum and differences of pairs of multiples of 10, 100 or 1000

Add or subtract any pair of two-digit numbers, including crossing the tens and 100 boundary, e.g. $47 + 58$, $91 - 35$

Add or subtract two-digit or three-digit multiples of 10, e.g. $120 - 40$, $140 + 150$, $370 - 180$
Add tens and ones separately, then recombine

Add or subtract a multiple of 10 and adjust, e.g. $56 + 29 = 56 + 30 - 1$, or $86 - 38 = 86 - 40 + 2$

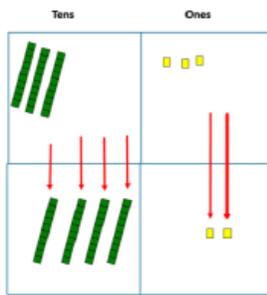
Key skills for

subtraction at Y4:

- Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 etc.
- Children select the most appropriate and efficient methods for given subtraction calculations.
- Estimate and use inverse operations to check answers.
- Solve addition and subtraction 2-step problems, choosing which operations and methods to use and why.
- Find 1000 more or less than a given number.
- Solve number and practical problems that involve the above, with increasingly large positive numbers

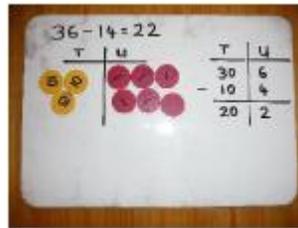
Column method without regrouping

Year 3 - up to three digits
Year 4 - up to 4 digits

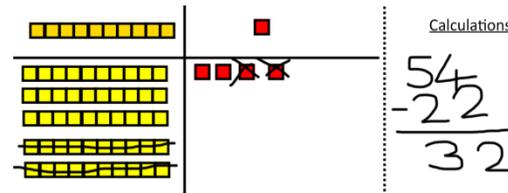


Use Base 10 to make the bigger number then take the smaller number away.

Show how you partition numbers to subtract. Again make the larger number first.



ENSURE YOU PARTITION AS THE FIRST STEP



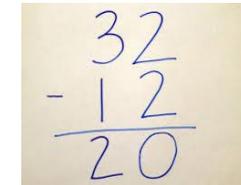
Draw the Base 10 or place value counters alongside the written calculation to help to show working.

$$47 - 24 = 23$$

$$\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$$

ENSURE YOU PARTITION AS THE FIRST STEP

This will lead to a clear written column



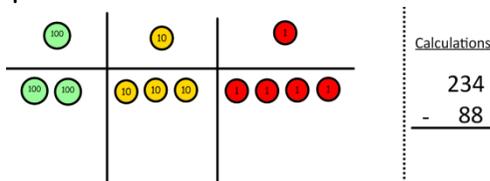
subtraction.

Column method with regrouping

Year 3 - up to three digits
Year 4 - up to 4 digits

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

Make the larger number with the place value counters



Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.

Draw the onto a grid and you have by the out as clearly showing the exchanges you make.

Hundreds	Tens	Ones
5	12	6
- 2	7	5
3	5	1

counters place value show what taken away crossing counters well as

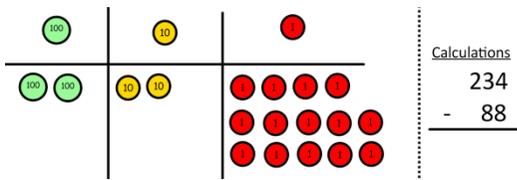
When confident, children can find their own way to record the exchange/regrouping.

Example 7 $74 - 27 = 47$

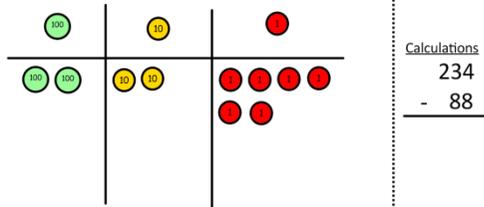
$$\begin{array}{r} 70 + 4 \\ - 20 + 7 \\ \hline \end{array} \quad \begin{array}{r} 60 \quad 14 \\ 70 + 4 \\ - 20 + 7 \\ \hline 40 + 7 \end{array}$$

Partition the numbers first within the method

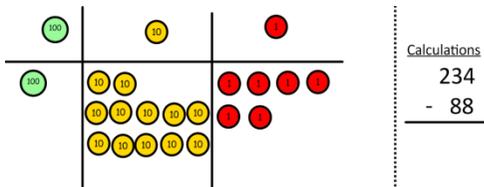
Children can start their formal written method by partitioning the number into clear place value columns.



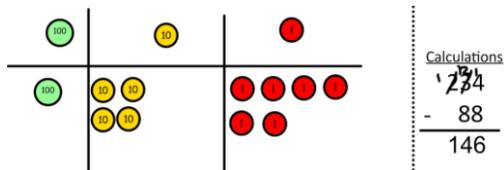
Now I can subtract my ones.



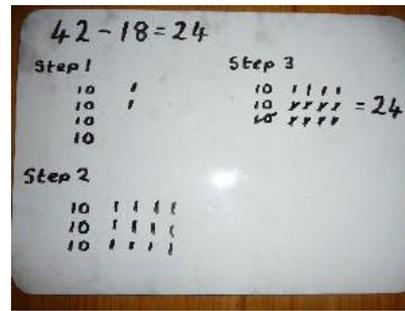
Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



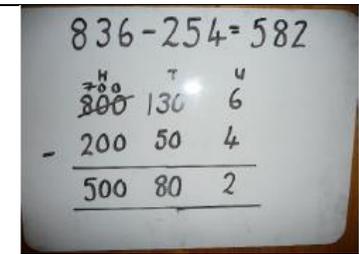
Now I can take away eight tens and complete my subtraction



Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.



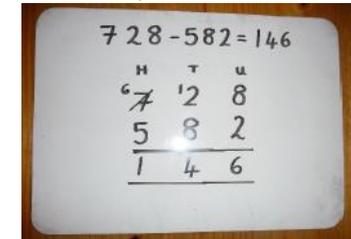
Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.



Example 8 $741 - 367 = 374$

$$\begin{array}{r} 700 + 40 + 1 \\ - 300 + 60 + 7 \\ \hline \end{array} \quad \begin{array}{r} 600 \quad 130 \quad 11 \\ 700 + 40 + 1 \\ - 300 + 60 + 7 \\ \hline 300 + 70 + 4 \end{array}$$

Moving forward the children use a more compact method.



This will lead to an understanding of subtracting any number including decimals.

$$\begin{array}{r} 5 \quad 12 \quad 1 \\ 2 \quad \cancel{6} \quad \cancel{3} \quad . \quad 0 \\ - \quad 2 \quad 6 \quad . \quad 5 \\ \hline 2 \quad 3 \quad 6 \quad . \quad 5 \end{array}$$

Year 5

Subtract numbers with up to five digits including money, measures and decimals with different numbers of decimal places

Year 6

Subtracting with increasingly large and more complex numbers and decimal values.

Key Vocabulary Year 5 and 6

equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse, **tenths**, **hundredths**, **decimal point**, **decimal**

Key skills for subtraction at Y5:

Subtract numbers mentally with increasingly large numbers .
Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy .
Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
Count forwards or backwards in steps of powers of 10 for any given number up to 1 million.

Links

[Moving to the compact column method of subtraction \(youtube\)](#)

Key skills for subtraction at Y6

Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
Read, write, order and compare numbers up to 10 million and determine the value of each digit
Round any whole number to a required degree of accuracy
Use negative numbers in context, and calculate intervals across zero.
Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before choosing how to calculate.

Mental 5

Use place value and number facts to add two or more friendly numbers including money and decimals
Add and subtract decimal numbers which are near multiples of 1 or 10 including money Add to the next 10 from a decimal number
To know what must be added to any four-digit number to make the next multiple of 10, 100 or 1000
To know what must be added to a decimal with units and tenths to make the next whole number
Add or subtract a pair or 2 digit numbers or 3 digit multiples of 10
Add or subtract a near multiple of 10 or 100 to any two-digit or three-digit number
Add or subtract a multiple of 10 or 100 and adjust
find 50%, 25% or 10% of whole numbers or quantities,

Mental 6

Add positive number to negative numbers (e.g calculate a rise in temp)
Add two 1-place decimal numbers or two 2-place decimal numbers less than 1 (4.5 + 6.5 or 0.74 +0.33)
Derive quickly and without difficulty, number bonds to 1000
Use number bonds to 1 and 10 to perform mental subtraction of any pair of one decimal place numbers
Addition and subtraction facts for multiples of 10 to 1000 and decimal numbers with one decimal place

Objective and Strategies

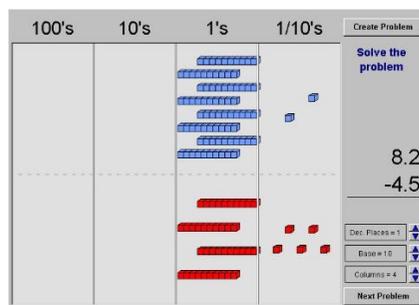
Concrete

Pictorial

Abstract

Column method with regrouping with larger numbers and decimals

Model decimal representations with diens to show the value of the decimal in the sme way as with whole numbers.



Move onto larger numbers and decimals but return to the concrete if children require.

Use of decimal place value counters as above in Year 3 and 4 but with decimals.



Drawing the concrete to show understanding.

Subtracting with larger integers

$$\begin{array}{r} \cancel{2}^{\text{2}} \cancel{1}^{\text{10}} \cancel{0}^{\text{10}} \cancel{8}^{\text{8}} \text{'6} \\ - \quad \quad 2 \quad 1 \quad 2 \quad 8 \\ \hline 2 \quad 8, \quad 9 \quad 2 \quad 8 \end{array}$$

Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.

$$\begin{array}{r} \cancel{7}^{\text{6}} \cancel{1}^{\text{10}} \cancel{6}^{\text{8}} \cancel{9}^{\text{8}} \cdot \text{'0} \\ - \quad \quad 3 \quad 7 \quad 2 \cdot 5 \\ \hline 6 \quad 7 \quad 9 \quad 6 \cdot 5 \end{array}$$

Using the compact column method to subtract money and measures, including decimals with different numbers of decimal places

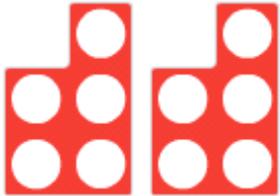
$$\begin{array}{r} \cancel{1}^{\text{1}} \cancel{0}^{\text{10}} \text{'5} \cdot \cancel{1}^{\text{1}} \cancel{1}^{\text{10}} \text{'9} \text{ kg} \\ - \quad \quad 3 \quad 6 \cdot 0 \quad 8 \quad \text{ kg} \\ \hline \quad \quad 6 \quad 9 \cdot 3 \quad 3 \quad 9 \text{ kg} \end{array}$$

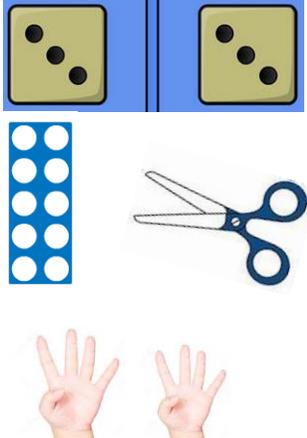
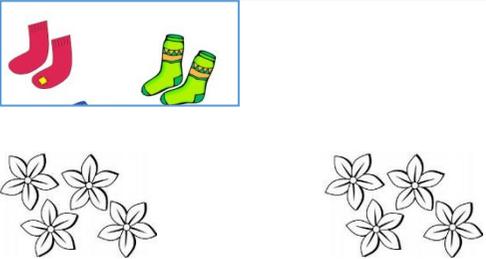
Multiplication

Multiplication Foundation stage

22-36 months:	30 - 50 months: Compares two groups of objects, saying when they have the same number.	40-60+ months:	Early Learning Goal: They solve problems, including doubling, halving and sharing.
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Objective and Strategies	Concrete	Pictorial	Abstract
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<p>30 - 50 months:</p> <p>Compares two groups of objects, saying when they have the same number.</p>			
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<p>Early Learning Goal:</p> <p>They solve problems, including doubling, halving and sharing.</p>			<div style="text-align: center;">  <p>4+4=</p> </div> <p>1+1=</p> <p>2+2=</p> <p>3+3=</p> <p>4+4=</p> <p>5+5=</p>
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Multiplication Year 1

Multiply with concrete objects, arrays and pictorial representations with support

Give children experience of counting equal group of objects in 2s,
5s and 10s.

Present practical problem solving activities involving counting equal sets or groups, as above.

Pictorial representations of arrays of 2, 5, and 10s. Ask children to find groups of and link to the number sentence

Key Vocabulary

groups of, lots of, times,
array, altogether, multiply,
count

Key skills for multiplication at Y1:

Count in multiples of 2, 5 and 10.
Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
Make connections between arrays, number patterns, and counting in twos, fives and tens. Begin to understand doubling using concrete objects and pictorial representations.

Links

Video Clips: 1-

<http://www.youtube.com/watch?v=YPWmOVt8vgw> Multiple Representations of Multiplication

Video Clip 2: <http://www.youtube.com/watch?v=VGkjjVfnGYI>

The commutative law for multiplication

<http://www.iboard.co.uk/activities/page/2/subject/maths/years/4-7>

Really high quality interactive games and demonstration tools

Mental

Counting in 10's, 5's and 2's

Know halves of even numbers to 20

Know doubles to 10

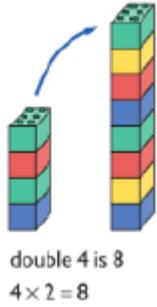
Objective and Strategies

Concrete

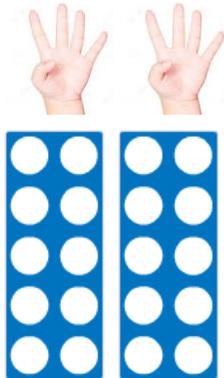
Pictorial

Abstract

Doubling

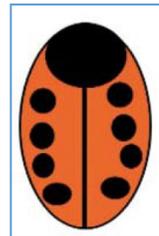
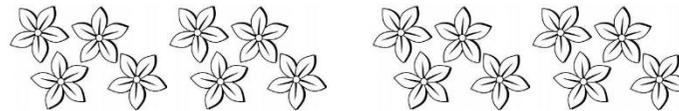


Use practical activities to show how to double a number.

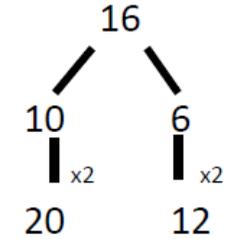


Draw pictures to show how to double a number.

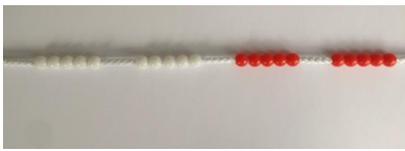
Double 4 is 8



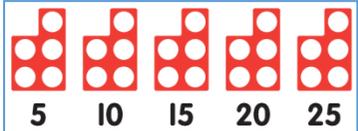
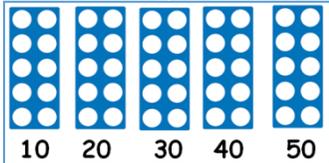
Partition a number and then double each part before recombining it back together.



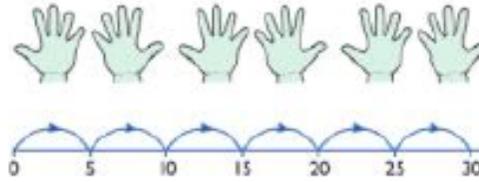
Counting in multiples



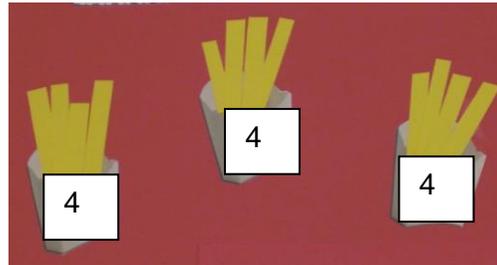
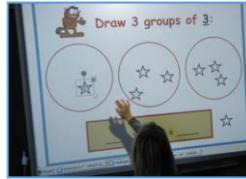
Count in multiples supported by concrete objects in equal groups.



Children learn to **count** in 2s, 5s and 10s off by heart.



Use a number line or pictures to continue support in counting in multiples.



If there are 4 chips in one packet. How many are there in 3 packets?

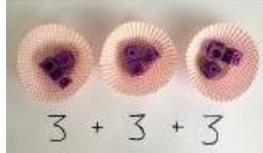
Count in multiples of a number aloud.

Write sequences with multiples of numbers.

2, 4, 6, 8, 10

5, 10, 15, 20, 25, 30

Repeated addition



$$3 + 3 + 3$$

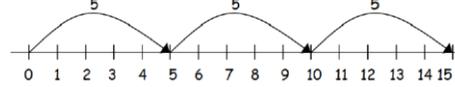


Use different objects to add equal groups.

There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?



2 add 2 add 2 equals 6



$$5 + 5 + 5 = 15$$



$$2 + 2 + 2 =$$

How many legs will 3 teddies have?

Write addition sentences to describe objects and pictures.



$$2 + 2 + 2 + 2 + 2 = 10$$

Multiplication Year 2

Multiply with concrete objects, arrays and pictorial representations with support

Give children experience of counting equal group of objects in 2s,
5s and 10s.

Present practical problem solving activities involving counting equal sets or groups, as above.

Pictorial representations of arrays of 2, 5, and 10s. Ask children to find groups of and link to the number sentence

Key Vocabulary

groups of, lots of, times,
array, altogether, multiply,
count

multiplied by, repeated
addition, column, row,
commutative, sets of, equal
groups, times as big as, once,
twice, three times...

Key skills for multiplication at Y2:

Count in steps of 2, 3 and 5 from zero, and in 10s
from any number.

Recall and use multiplication facts from the 2, 5
and 10 multiplication tables, including recognising
odds and evens.

Write and calculate number statements using the x
and = signs.

Show that multiplication can be done in any order
(commutative).

Solve a range of problems involving multiplication,
using concrete objects, arrays, repeated addition,
mental methods, and multiplication facts.

Pupils use a variety of language to discuss and
describe multiplication.

Links

Video clips:

[Teaching for understanding of multiplication facts](#) (youtube)

[Practical multiplication and the commutative law](#) (youtube)

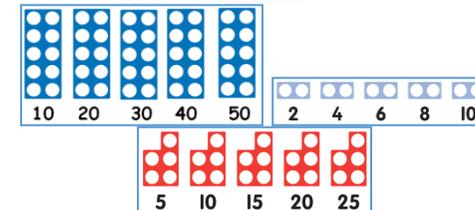
<http://www.snappymaths.com/multiplication/earlymult/earlymult.htm>

Really nice early multiplication resources for whiteboard and
worksheets to print.

<http://www.iboard.co.uk/activities/page/2/subject/maths/years/4-7>

Really high quality interactive games and demonstration tools

Mental



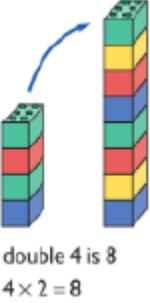
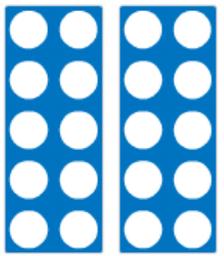
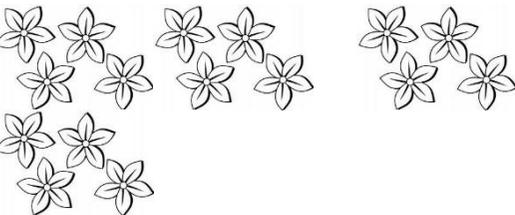
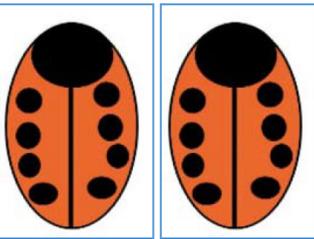
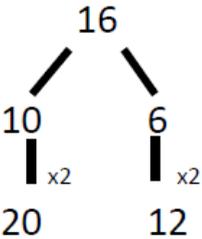
They learn the 2, 5 and 10 times table facts by heart to the 12th
multiple

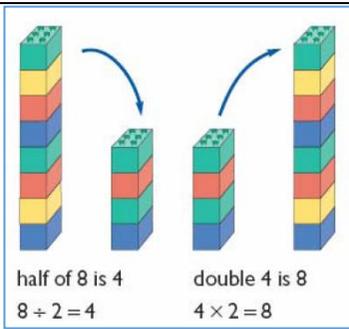
They learn doubles of numbers to 10 off by heart.

To know doubles of all numbers to 20, e.g. double 13, and corresponding
halves

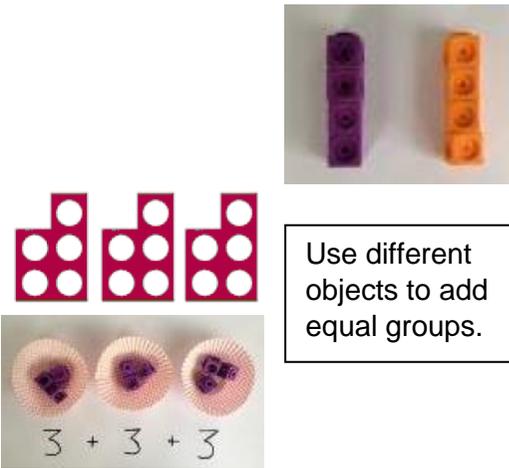
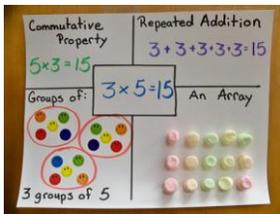
To be able to recall doubles of multiples of 10 to 100, e.g. double 40, and
corresponding halves

To recall multiplication facts for the 2, 5 and 10 times-tables, and
corresponding division facts

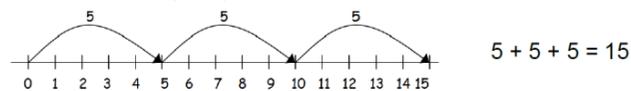
Objective and Strategies	Concrete	Pictorial	Abstract
<p>Doubling</p>	 <p>double 4 is 8 $4 \times 2 = 8$</p> <p>Use practical activities to show how to double a number.</p>   <p>Children learn how to find doubles and halves. They know that doubling is the inverse of halving.</p>	<p>Draw pictures to show how to double a number.</p> <p>Double 4 is 8</p>    	<p>Partition a number and then double each part before recombining it back together.</p>  <p>Children learn how to find doubles and halves. They know that doubling is the inverse of halving. $8 \times 2 = 16$ $16 \div 2 = 8$</p>



Repeated addition



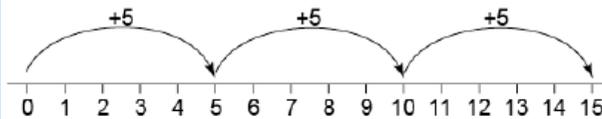
2 add 2 add 2 equals 6



$2 + 2 + 2 =$

How many legs will 3 teddies have?

5×3

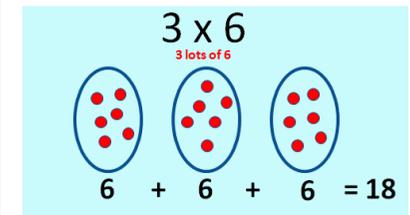


Children show jumps on a number line.

Write addition sentences to describe objects and pictures.



Children make groups to calculate multiplications. They record the repeated addition underneath:



They can then move to just recording the repeated addition:

7×4
 $4 + 4 + 4 + 4 + 4 + 4 + 4 = 28$

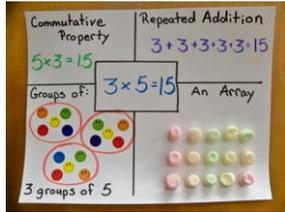
1 bar of chocolate costs 5p.
 How much will it cost to buy 4 bars?

$4 \times 5p$

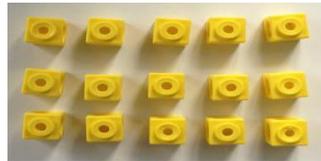
5p

$5p + 5p + 5p + 5p$

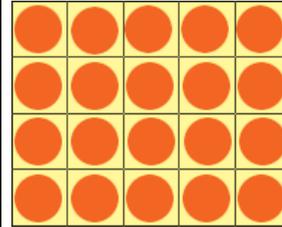
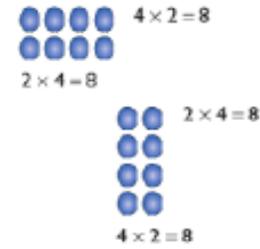
Arrays- showing commutative multiplication



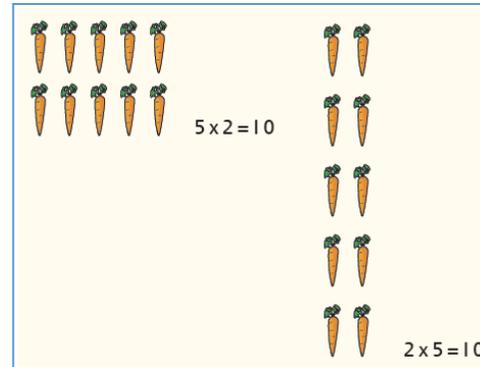
Create arrays using counters/ cubes to show multiplication sentences.



Draw arrays in different rotations to find **commutative** multiplication sentences.



Link arrays to area of rectangles.



Use an array to write multiplication sentences and reinforce repeated addition.



$$5 + 5 + 5 = 15$$

$$3 + 3 + 3 + 3 + 3 = 15$$

$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

Multiplication Year 3

Multiply 2-digits by a single digit number

Partition numbers into tens and units

Multiply multiples of ten by a single digit (e.g. 20×4) using their knowledge of multiplication facts and place value

Recall and work out multiplication facts in the 2, 3, 4, 5, 8 and 10 times tables

Work out multiplication facts not known by repeated addition or other taught mental strategies (e.g. by commutative law, working out near multiples and adjusting, using doubling etc.) Strategies to support this are repeated addition using a number line, bead bars and arrays

Multiplication Year 4

Multiply 2 and 3-digits by a single digit, using all multiplication tables up to 12×12

Approximate before they calculate, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer.

Record an approximation to check the final answer against.

Multiply multiples of ten and one hundred by a single-digit, using their multiplication table knowledge.

Recall all times tables up to 12×12

Key Vocabulary

groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times, _times as big as, once, twice, three times...

partition, grid method, multiple, product, tens, units, value

Key skills for subtraction at Y3:

- Recall and use multiplication facts for the **2, 3, 4, 5, 8 and 10** multiplication tables, and multiply multiples of 10.
- Write and calculate number statements using the multiplication tables they know, including **2-digit \times single-digit**, drawing upon mental methods, and progressing to reliable written methods.
- Solve multiplication problems, including missing number problems.

Links

Video clips:

[Teaching the grid method as an interim step](#)

(partitioning and counters to introduce grid)

Key Vocabulary

groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, groups of, sets of, lots of, equal groups, times, multiply, times as big as, once, twice, three times... partition, grid method, total, multiple, product, sets of, **inverse**

Key skills for subtraction at Y4:

- Count in multiples of 6, 7, 9, 25 and 1000
- Recall multiplication facts for **all multiplication tables up to 12×12** .
- Recognise place value of digits in up to 4-digit number
- Use place value, known facts and derived facts to multiply mentally, e.g. multiply by 1, 10, 100, by 0, or to multiply 3 numbers.
- Use commutativity and other strategies mentally $3 \times 6 = 6 \times 3$,

Mental

3
6
9
4
8
12

To recall multiplication facts for the 2, 3, 4, 5, 8 and 10 times-tables, and corresponding division facts
 Double numbers up to 50
 Know pairs with each total to 20
 Recall fact families
 Develop mental strategies using commutativity (e.g. $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$)

- Develop mental strategies using commutativity (e.g. $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$)
- Solve simple problems in contexts, deciding which operations and methods to use.
- Develop efficient mental methods to solve a range of problems e.g using commutativity

Mental

9 18 27

To recall multiplication facts to 12×12 and the corresponding division facts
 Double any two-digit number, e.g. double 39
 Double any multiple of 10 or 100 and halve the corresponding multiples of 10 and 100
 Multiply and divide numbers to 1000 by 10 and then 100 (whole-number answers),
 Multiply a multiple of 10 to 100 by a single-digit number
 Multiply two multiples of 10.

- $2 \times 6 \times 5 = 10 \times 6$, $39 \times 7 = 30 \times 7 + 9 \times 7$.
- Solve problems with increasingly complex multiplication in a range of contexts.
 - Count in multiples of 6, 7, 9, 25 and 1000
 - Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)

Objective and Strategies	Concrete	Pictorial	Abstract
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Grid Method

Show the link with arrays to first introduce the grid method.

x	10	3
4		4 rows of 10 4 rows of 3

Move on to using Base 10 to move towards a more compact method.

4 rows of 13

Children can represent the work they have done with place value counters in a way that they understand.

Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

x	30	5
7	210	35

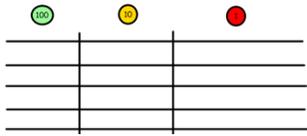
$210 + 35 = 245$

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

x	T	U
	■■■■	■ ■
	■■■■	■ ■
	■■■■	■ ■
	■■■■	■ ■

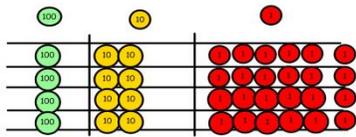
Move on to place value counters to show how we are finding groups of

a number. We are multiplying by 4 so we need 4 rows.



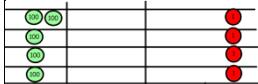
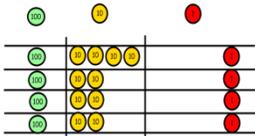
Calculations
4 x 126

Fill each row with 126.



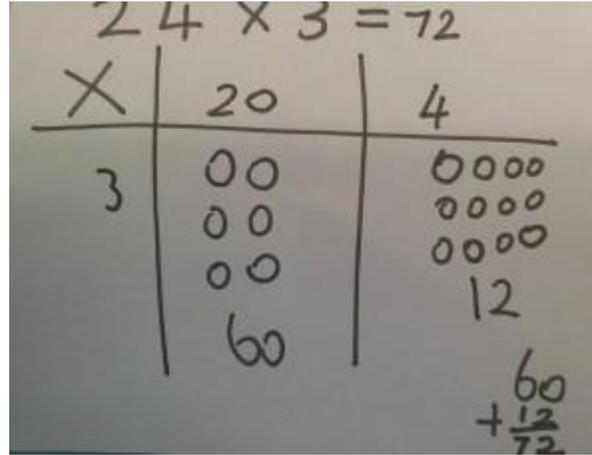
Calculations
4 x 126

Add up each column, starting with the ones making any exchanges needed.



Then you have your answer.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



	10	8
10	100	80
3	30	24

x	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

Using commutativity

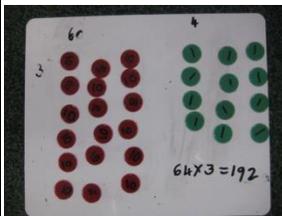
4 x 12 x 5 =
4 x 5 x 12 =
20 x 12 = 240

Derive related facts
(facts for free)

2 x 3 = 6 20 x 30 = 60
3 x 2 = 6 30 x 20 = 60
6 ÷ 3 = 2 60 ÷ 30 = 20
6 ÷ 2 = 3 60 ÷ 20 = 30

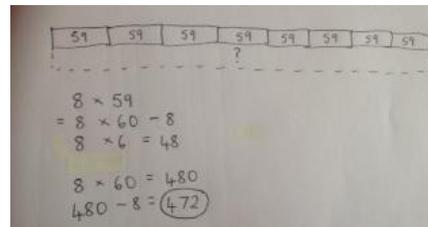
Column multiplication

Children can continue to be supported by place value counters at the stage of multiplication.



It is important at this stage that they always multiply the ones first and note

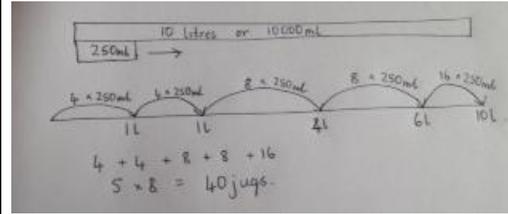
Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.



	T	U	
		4	
x		3	
	1	2	
	T	U	
		3	7
x			4
		2	8 (7 x 4)
	1	2	0 (30 x 4)

Start with short multiplication, reminding the children about lining

down their answer followed by the tens which they note below.



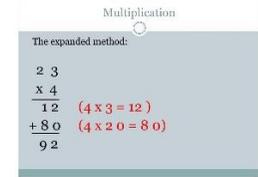
x	300	20	7
4	1200	80	28



	3	2	7
x			4
	1	3	0
		1	2
			8

Pupils could be asked to work out a given calculation using the grid, and then compare it to „your“ column method. What are the similarities and differences? Unpick the steps and show how it reduces the steps

up their numbers clearly in columns.
If it helps, children can write out what they are solving next to their answer.



Short multiplication

24 x 6 becomes

$$\begin{array}{r} 24 \\ \times 6 \\ \hline 144 \\ \hline 2 \end{array}$$

Answer: 144

342 x 7 becomes

$$\begin{array}{r} 342 \\ \times 7 \\ \hline 2394 \\ \hline 21 \end{array}$$

Answer: 2394

Multiplication Year 5

Multiply up to 4-digits by 1 or 2 digits.

Introduce by comparing a grid method calculation to a short multiplication method, to see how the steps are related, but notice how there are less steps involved in the column method (see video).

Children need to be taught to approximate first, e.g. for 72×38 , they will use rounding: 72×38 is approximately $70 \times 40 = 2800$, and use the approximation to check the reasonableness of their answer against.

Multiplication Year 6

Short and long multiplication as in Y5, and multiply single digit numbers with up to 2d.p by a single digit.

Use rounding and place value to make approximations before calculating and use these to check answers against.

Use short multiplication (see Y5) to multiply numbers with more than 4-digits by a single digit; to multiply money and measures, and to multiply decimals with up to 2d.p. by a single digit.

Use long multiplication (see Y5) to multiply numbers with at least 4 digits by a 2-digit number.

Key Vocabulary

groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, _times as big as, once, twice, three times..., partition, grid method, total, multiple, product, inverse, square, factor, integer, decimal, short/long multiplication, _carry'

Key skills for

multiplication at Y5:

- Identify multiples and factors, using knowledge of **multiplication tables to 12×12** .
- Solve problems where larger numbers are decomposed into their factors
- Multiply and divide integers and decimals by 10, 100 and 1000
- Recognise and use square and cube numbers and their notation
- Solve problems involving combinations of operations, choosing and using calculations and methods appropriately.

Links

Video Links

[Demonstration of long multiplication](#)
[Moving from grid method to a compact method](#)
[Reinforcing rapid times table recall:](#)

Key Vocabulary

groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times... partition, grid method, total, multiple, product, inverse, square, factor, integer, decimal, short / long multiplication, „carry“, **tenths, hundredths, decimal**

Key skills for multiplication

at Y6:

Recall multiplication facts for all times tables up to **12×12 (as Y4 and Y5)**.
Multiply multi-digit numbers, up to 4-digit \times 2-digit using long multiplication.
Perform mental calculations with mixed operations and large numbers. Solve multi-step problems in a range of contexts, choosing appropriate combinations of operations and methods.
Estimate answers using round and approximation and determine levels of accuracy.
Round any integer to a required degree of accuracy.

Mental

Double and halve money by partitioning (Half of £75.40 = Half of £75 (37.50) plus half of 40p)
 Recall times tables and division facts for all tables up to 12 .
 To know square numbers and square roots up to 144.
 To recall doubles and halves of decimals, e.g. half of 5.6, double 3.4
 To multiply and divide whole numbers and decimals by 10, 100 or 1000
 To multiply pairs of multiples of 10, e.g. 60×30 , and a multiple of 100 by a single digit number, e.g. 900×8

Mental

Halve and double decimal numbers with up to 2 places using partitioning
 Know by heart all multiplication and division facts up to 12×12 .
 Use rounding in mental multiplication (34×19 as $(20 \times 34) - 34$)
 Use doubling and halving as a mental division and multiplication strategy
 Use divisibility tests to aid mental calculation
 Use place value and number facts in mental multiplication ($40,000 \times 6 = 240,000$)
 To recall square numbers to 12×12
 To recall square numbers of the corresponding multiples of 10
 To multiply pairs of multiples of 10 and 100, e.g. 50×30 , 600×20
 To multiply and divide any whole or decimal place numbers by 10, 100, 1000

Objective and Strategies

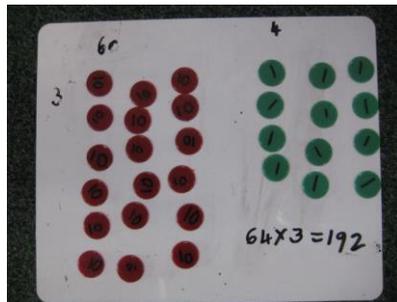
Concrete

Pictorial

Abstract

Column multiplication

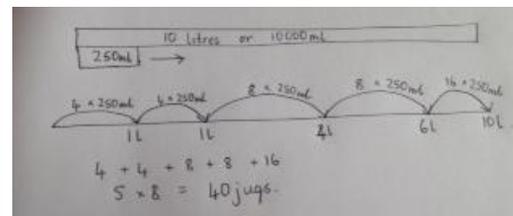
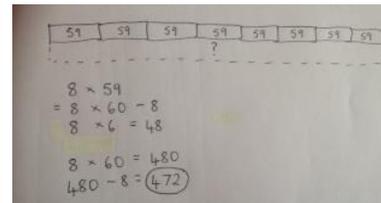
Children can continue to be supported by place value counters at the stage of multiplication.



It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

Place value counters can also be used with decimal numbers using decimal place value counters in the same way as above.

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.



		T	U
		3	7
x		1	2
		4	8

Recap short multiplication

Start with long multiplication, reminding the children about lining up their numbers clearly in columns.
 If it helps, children can write out what they are solving next to their answer.

$$\begin{array}{r}
 32 \\
 \times 24 \\
 \hline
 128 \quad (4 \times 2) \\
 640 \quad (4 \times 30) \\
 \hline
 768
 \end{array}$$

		T	U	
		3	7	
x		2	4	
		2	8	(7 x 4)
	1	2	0	(30 x 4)
	1	4	0	(7 x 20)
	6	0	0	(30 x 20)
	8	8	8	

Remove brackets when ready

	T	U
	3	7
x	2	4
	2	8
	1	2
	1	4
	6	0
	8	8

	7	4
x	6	3
	1	2
	2	1
	2	4
+	4	2
	4	6
	6	6
	2	3
	1	

This moves to the more compact method.

	1	3	4	2
x		1	8	
	1	3	4	2
	1	0	7	3
	2	4	1	5
	6	2		

Year six move to decimals

$$6 \times 1.67 =$$

1	.	6	7
1	4	4	6
			x
1	0	.	0
		2	

3	.	1	9
x	8		
2	5	.	5
		2	

Remind children that the single digit belongs in the units column.

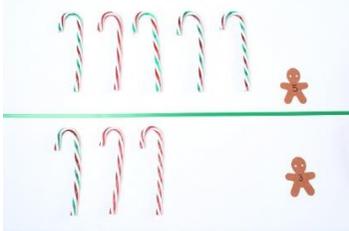
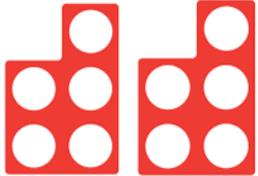
Line up the decimal points in the question and the answer.

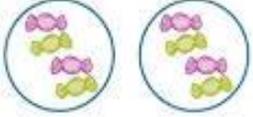
This works well for multiplying money (£, p) and other measures.

Division

Division Foundation stage

22-36 months: Begins to make comparisons between quantities.	30 – 50 months: Compares two groups of objects, saying when they have the same number.	40 - 60+ months: Begins to identify own mathematical problems based on own interests and fascinations.	Early Learning Goal: They solve problems, including doubling, halving and sharing.
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Objective and Strategies	Concrete	Pictorial	Abstract
22-36 months: Begins to make comparisons between quantities.			
30 – 50 months: Compares two groups of objects, saying when they have the same number.			
40 - 60+ months: Begins to identify own mathematical problems based on own interests and fascinations.			

Early Learning Goal: They solve problems, including doubling, halving and sharing.	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> Sharing problems  </div> <div style="text-align: center;"> Halving  </div> </div>	<div style="text-align: center;">  $4+4=$ </div> <div style="margin-top: 10px;"> $1+1=$ $2+2=$ $3+3=$ $4+4=$ $5+5=$ </div>
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Division Year 1
Group and share small quantities

Using objects, diagrams and pictorial representations to solve problems involving both grouping and sharing.

Division Year 2
Group and share, using the ÷ and = sign

Use objects, arrays, diagrams and pictorial representations, and grouping on a number line

Key Vocabulary
 share, share equally, one each, two each..., group, groups of, lots of, array

- Key skills for division at Y1:**
- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher
 - Through grouping and sharing small quantities, pupils begin to understand, division, and finding
 - Simple fractions of objects, numbers and quantities.
 - They make connections between arrays, number patterns, and counting in twos, fives and tens.

Links

Video Clip: 1

<https://www.ncetm.org.uk/resources/43589>

Sharing and Grouping

<http://www.iboard.co.uk/iwb/Equal-Bees-421>

all iboard activities are really good. In this one the bees fly into equal groups.

Key Vocabulary
 share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over

- Key skills for division at Y2:**
- Count in steps of 2, 3, and 5 from 0
 - Recall and use multiplication and division facts for the **2, 5 and 10** multiplication tables, including recognising odd and even numbers.
 - Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the \times , \div and $=$ signs.
 - Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
 - Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

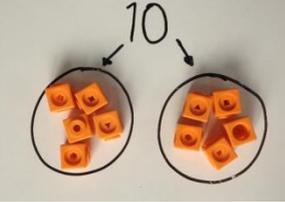
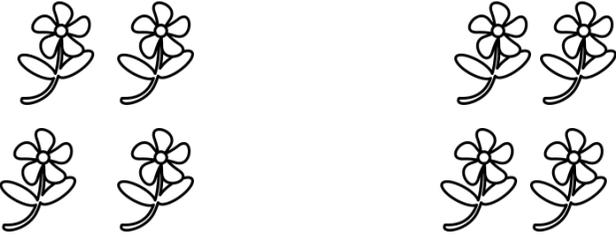
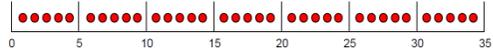
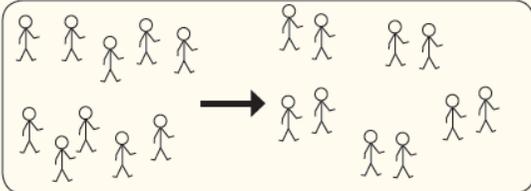
Mental

Counting in 10's, 5's and 2's

Know halves of even numbers to 20

Know doubles to 10

- Mental**
- I can recall all the division facts to $24 \div 2$
 - I can recall all the division facts to $60 \div 5$
 - I can recall all the division facts to $120 \div 10$
- To know doubles of all numbers to 20 and corresponding halves , e.g. double 13, and corresponding halves
- To be able to recall doubles of multiples of 10 to 100, e.g. double 40, and corresponding halves

Objective and Strategies	Concrete	Pictorial	Abstract
<p>Sharing objects into groups</p> <p>(Halving)</p>	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>  <p>$8 \div 4 = 2$</p> 	 <p>$20 \div 2 = 10$</p> <p>Children use pictures or shapes to share quantities.</p>  <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;"> $8 \div 2 = 4$ </div>	<p>Share 9 buns between three people.</p> <p>$9 \div 3 = 3$</p> <p>Make clear links with multiplication</p>  <p>$3 \times 2 = 6$</p> <p>$6 \div 3 = 2$</p>
<p>Division as grouping</p>	<p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p> 	 <p><i>There are 10 people. How many groups of 2 can I make?</i></p>	<p>$28 \div 7 = 4$</p> <p>Divide 28 into 7 groups. How many are in each group?</p>



Making 10 using 2's or 5's



How many 5's make 10? $10 \div 5 =$

How many 2's make 10? $10 \div 2 =$

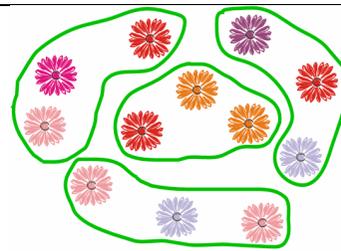


Using Numicon

$$12 \div 3 = 4$$

"How many 3s make 12?"

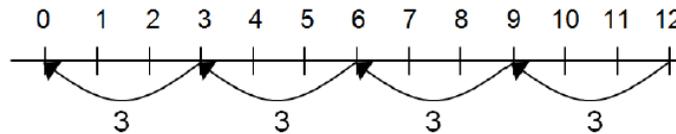
"12 divided into 3s"



There are 12 flowers.

How many groups of 3 can you make?

Use a number line to show jumps in groups. The number of jumps equals the number of groups.



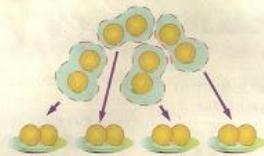
Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.



$$20 \div 5 = ?$$

$$5 \times ? = 20$$

2. Let us divide by grouping.



8 oranges are put into groups of two.
We get 4 groups of two.

We write the number sentence as:

$$8 \div 2 = 4$$

Division as Sharing Equally and Grouping

1. Let us divide by sharing equally.

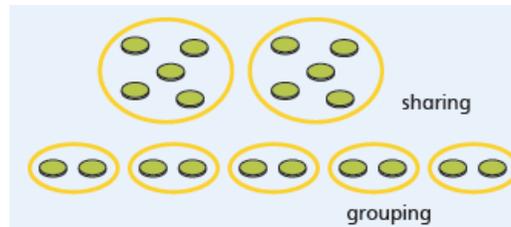
There are 8 apples.
We want to share them equally.

Kavita
Ana

We get 4 apples each.
We write the number sentence as
 $8 \div 2 = 4$
We read it as
eight divided by two equals four.
To share equally means to divide.
÷ stands for division.

$$10 \div 2 = 5$$

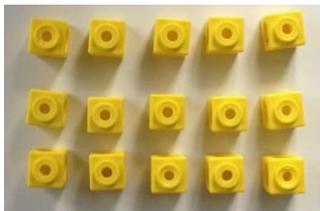
Share 10 between 2 groups



How many **groups** of 2 make 10?

Division
within
arrays

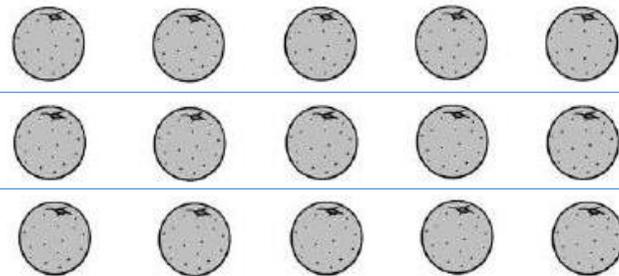
Year 2



Link division
to
multiplication
by creating
an array and
thinking

about the number sentences that can
be created.

Eg $15 \div 3 = 5$ $5 \times 3 = 15$
 $15 \div 5 = 3$ $3 \times 5 = 15$



Draw an array and use lines to split the array into
groups to make multiplication and division sentences.

Find the inverse of multiplication
and division sentences by creating
four linking number sentences.

$$7 \times 5 = 35$$

$$5 \times 7 = 35$$

$$35 \div 7 = 5$$

$$35 \div 5 = 7$$

Division Year 3

Divide 2-digit numbers by a single digit using the times tables they already know
(where there is no remainder in the final answer)

Division Year 4

Divide up to 3-digit numbers by a single digit.

Key Vocabulary

share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple

Key skills for division at Y3:

- Recall and use multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables (through doubling, connect the 2, 4 and 8s).
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- Solve problems, in contexts, and including missing number problems, involving multiplication and division.
- Pupils develop efficient mental methods, for example, using multiplication and division facts (e.g. using $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts ($30 \times 2 = 60$, so $60 \div 3 = 20$ and $20 = 60 \div 3$).
- Pupils develop reliable written methods for division, starting with calculations of 2-digit numbers by 1-digit numbers and progressing to the formal written method of short division.

Links

Video Clip: 1

<https://www.ncetm.org.uk/resources/43589>

Sharing and Grouping

Moving towards a written algorithm: 2 Using place value counters

<https://www.ncetm.org.uk/resources/43589>

<http://www.iboard.co.uk/iwb/Equal-Bees-421>

all iboard activities are really good. In this one the bees fly into equal groups.

<http://www.youtube.com/watch?v=KGMf314LUcO>
- really useful YouTube tutorial for short division

Division with remainders

<http://www.taw.org.uk/lic/itp/reminders.html>

Key Vocabulary

share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor

Key skills for division at Y4:

- Recall multiplication and division facts for all numbers up to 12×12 .
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1.
- Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number
- Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example $200 \times 3 = 600$ so $600 \div 3 = 200$
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

Mental

Derive related division facts
(facts for free - fact families)

$$\begin{array}{ll} 2 \times 3 = 6 & 2 \times 30 = 60 \\ 3 \times 2 = 6 & 30 \times 2 = 60 \\ 6 \div 3 = 2 & 60 \div 30 = 2 \\ 6 \div 2 = 3 & 60 \div 2 = 30 \end{array}$$

Recall all division facts for 2, 5, 10, 3, 4, 8 times tables.

To recall multiplication facts for the 2, 3, 4, 5, 8 and 10 times-tables, and corresponding division facts

Halve even numbers up to 100, halve odd numbers to 20.

Mental

Derive related division facts
(facts for free - fact families)

$$\begin{array}{ll} 2 \times 3 = 6 & 2 \times 30 = 60 \\ 3 \times 2 = 6 & 30 \times 2 = 60 \\ 6 \div 3 = 2 & 60 \div 30 = 2 \\ 6 \div 2 = 3 & 60 \div 2 = 30 \end{array}$$

Recall all division facts to 12×12

Halve any even number to 200

Multiply and divide numbers to 1000 by 10 and then 100
(whole-number answers), e.g. 325×10 , 42×100 , $120 \div 10$,
 $600 \div 100$, $850 \div 10$

Objective and
Strategies

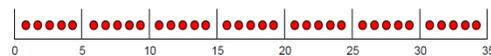
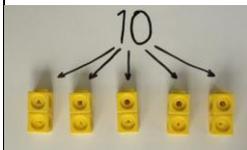
Concrete

Pictorial

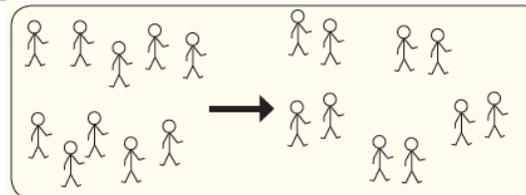
Abstract

Division as
grouping

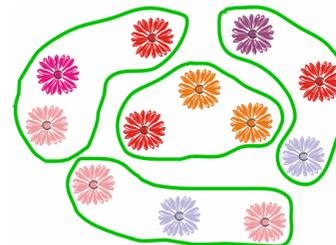
Divide quantities into equal groups.
Use cubes, counters, objects or place value
counters to aid
understanding.



Making 10 using 2's or 5's

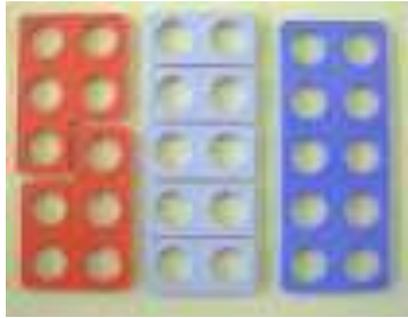


There are 10 people. How many groups
of 2 can I make?



$$28 \div 7 = 4$$

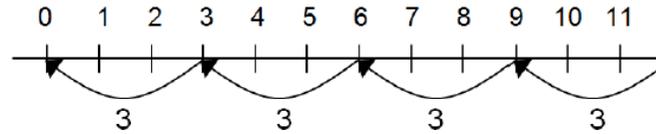
Divide 28 into 7 groups. How many
are in each group?



How many 5's make 10? $10 \div 5 =$
 How many 2's make 10? $10 \div 2 =$

There are 12 flowers.
 How many groups of 3 can you make?

Use a number line to show jumps in groups. The number of jumps equals the number of groups.



Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.

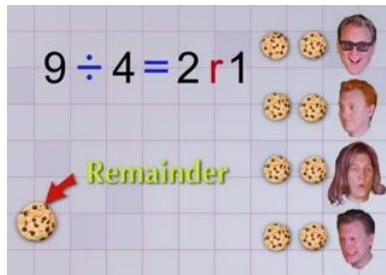
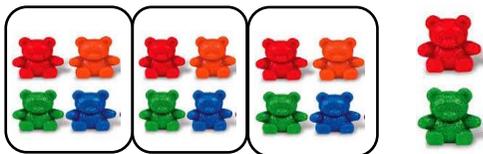


$$20 \div 5 = ?$$

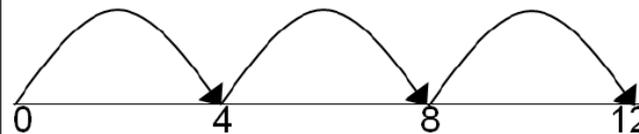
$$5 \times ? = 20$$

Division with a remainder

$14 \div 3 =$
 Divide objects between groups and see how much is left over



Jump forward in equal jumps on a number line



then see how many more you need to jump to find a remainder.

Draw dots and group them to divide an amount and clearly show a remainder.



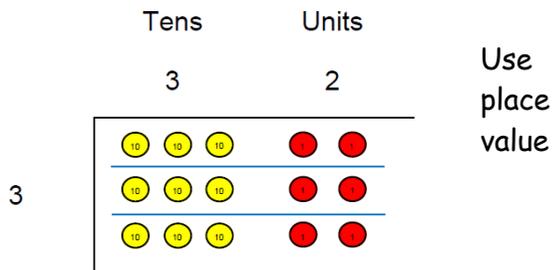
Complete written divisions and show the remainder using r.

$$29 \div 8 = 3 \text{ REMAINDER } 5$$

\uparrow \uparrow \uparrow \uparrow
 dividend divisor quotient remainder

Children do not move onto short division until they have the full conceptual knowledge of remainders

Short division
(Year 3 without remainders
Year 4 with remainders)

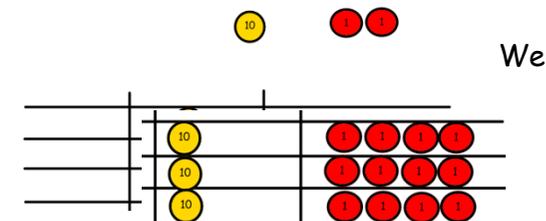


Use place value

counters to divide using the bus stop method alongside

Calculations
 $42 \div 3 = 14$

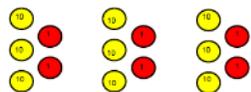
biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



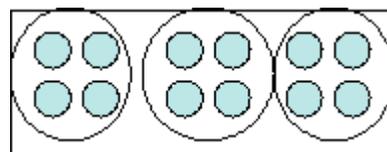
exchange this ten for ten ones and then share the ones equally among the groups.

We look how much in 1 group so the answer is 14.

$$96 \div 3 = 32$$

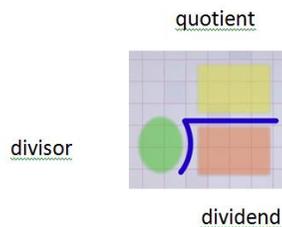


Students can continue to use drawn diagrams with dots or circles to help them divide

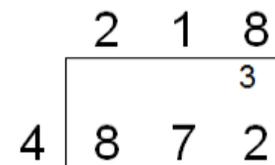
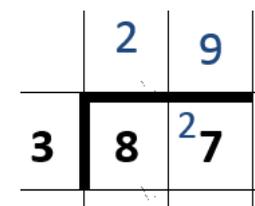


numbers into equal groups.

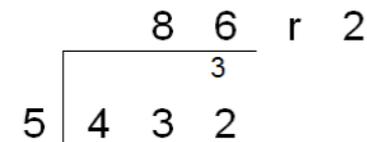
Encourage them to move towards counting in multiples to divide more efficiently.



Begin with divisions that divide equally with no remainder.



Move onto divisions with a remainder.



Division Year 5

Divide up to 4 digits by a single digit, including those with remainders

Division Year 6

Divide at least 4 digits by both single-digit and 2-digit numbers (including decimal numbers and quantities)

Key Vocabulary

Y5 and 6

share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, „carry“, remainder, multiple, divisible by, factor, inverse, quotient, prime number, prime factors, composite number (non-prime)

Key skills for division at Y5:

- Recall multiplication and division facts for all numbers up to 12×12 (as in Y4).
- Multiply and divide numbers mentally, drawing upon known facts.
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two number.
- Solve problems involving multiplication and division where larger numbers are decomposed into their factors.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Work out whether a number up to 100 is prime, and recall prime numbers to 19.
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Use multiplication and division as inverses.
- Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. $98 \div 4 = 24 \text{ r } 2 = 24\frac{1}{2} = 24.5 \approx 25$).
- Solve problems involving combinations of all four operations, including understanding of the equals sign, and including division for scaling by different fractions and problems involving simple rates.

Links

Moving towards a written algorithm: 2
Using place value counters

<https://www.ncetm.org.uk/resources/43589>

Division with exchange
<https://www.ncetm.org.uk/resources/43589>

<http://www.youtube.com/watch?v=KGMf314LUc0> - really useful
YouTube tutorial for short division

<http://www.youtube.com/watch?v=L6qBQrUYuq4> - really useful
YouTube tutorial for long division

Key skills for division at Y6:

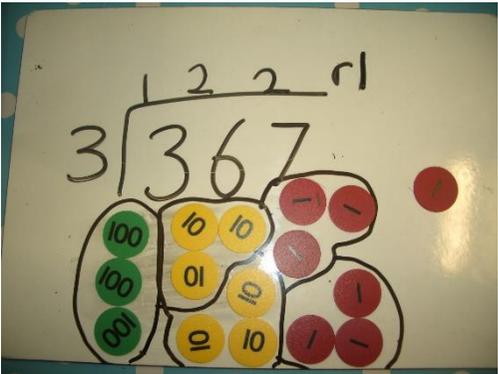
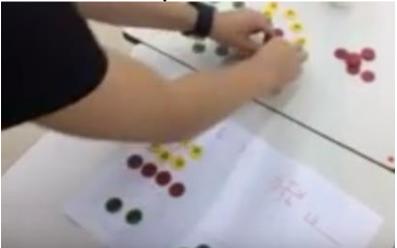
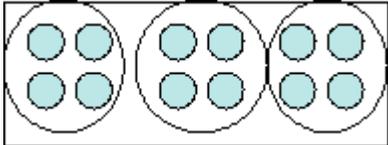
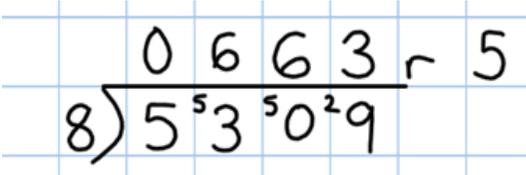
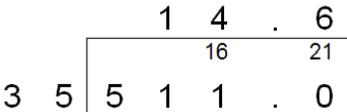
- Recall and use multiplication and division facts for all numbers to 12×12 for more complex calculations
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Use short division where appropriate.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers.
- Solve problems involving all 4 operations.
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem.
- Use written division methods in cases where the answer has up to two decimal places.
- Solve problems which require answers to be rounded to specified degrees of accuracy.

Mental

Double and halve money by partitioning (Half of £75.40 = Half of £75 (37.50) plus half of 40p)
 Recall times tables and division facts for all tables up to 12 .
 To know square numbers and square roots up to 144.
 To recall doubles and halves of decimals, e.g. half of 5.6, double 3.4
 To multiply and divide whole numbers and decimals by 10, 100 or 1000

Mental

Halve and double decimal numbers with up to 2 places using partitioning
 Know by heart all multiplication and division facts up to 12×12 .
 Use doubling and halving as a mental division and multiplication strategy
 Use divisibility tests to aid mental calculation
 Use place value and number facts in mental multiplication ($40,000 \times 6 = 240,000$)
 To multiply and divide any whole or decimal place numbers by 10, 100, 1000

Objective and Strategies	Concrete	Pictorial	Abstract
<p>Short division</p>	<p>Use place value counters to divide using the bus stop method alongside $367 \div 3 =$</p>  <p>The video link below shows clearly the process of using place value counters to show the process of dividing by a one digit number using the long division method but could easily be adapted to suit bus stop .</p> 	<p>Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups representing place value counters pictorially</p>  <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	<p>Short division with remainders: Now that pupils are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context,</p>  <p>where pupils consider the meaning of the remainder and <u>how</u> to express it</p> <p>The answer to $5309 \div 8$ could be expressed as 663 and five eighths, 663 r 5, as a decimal, or rounded as appropriate to the problem involved. Finally move into decimal places to divide the total accurately.</p> 

<https://www.youtube.com/watch?v=LbxbDpnakoY>

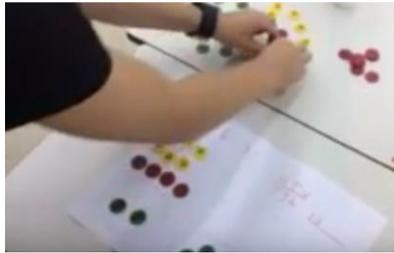
$$\begin{array}{r} 0812.125 \\ 8 \overline{)6497.000} \end{array}$$

	1	4	1	5	1/3
3	4	¹ 2	4	¹ 6	
	1	4	1	5	.3
3	4	¹ 2	4	¹ 6	

Long Division Year 6

Concrete and pictorial demonstrations with short division must be secure to move onto the formal long division algorithm

The video link below shows clearly the process of using placevalue counters to show the process of dividing by a one digit number using the long division method



<https://www.youtube.com/watch?v=LbxbDpnakoY>



Abstract

432 ÷ 15 becomes

$$\begin{array}{r} 28 \\ 15 \overline{)432} \\ \underline{30} \\ 132 \\ \underline{120} \\ 12 \end{array}$$

Explanation

$$15 \times 20 = 300$$

$$432 - 300 = 132 \text{ (this remains)}$$

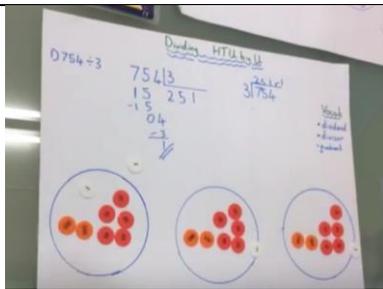
$$15 \times 8 = 120$$

$$132 - 120 = 12 \text{ (this remains)}$$

Answer: 28 remainder 12

This video clearly illustrates how long division can be illustrated with images of base ten. Equipment could be used to aid understanding or at the concrete stage





<https://www.youtube.com/watch?v=T8TtAnOxiV8>

<https://www.youtube.com/watch?v=8IXAqXGDMXw>

Some children may still need written explanations

<u>125</u>	(Explanations)		
4) 500			
<u>4</u>	(4 × 1 = 4)	2191	
10	(5 - 4 = 1)	4 8764	216 4536
<u>8</u>	(4 × 2 = 8)	8 ↓	432 ↓
20	(10 - 8 = 2)	07 ↓	216
<u>20</u>	(4 × 5 = 20)	4 ↓	216
0	(20 - 20 = 0)	36 ↓	0
		36 ↓	17 r 19
		04	31 ↓
		4	236
		0	217
			19

Answer as a fraction

432 ÷ 15 becomes

$$\begin{array}{r}
 28 \\
 15 \overline{) 432} \\
 \underline{30} \\
 132 \\
 \underline{120} \\
 12
 \end{array}$$

$$\frac{12}{15} = \frac{4}{5}$$

Answer: 28 ⁴/₅

Answer as a decimal

432 ÷ 15 becomes

$$\begin{array}{r}
 28.8 \\
 15 \overline{) 432.0} \\
 \underline{30} \\
 132 \\
 \underline{120} \\
 120 \\
 \underline{120} \\
 0
 \end{array}$$

Answer: 28.8

Explanation

15 x 2 = 30

43 - 30 = 13 (this remains)

Bring down the next digit (this remains)

15 x 8 = 120

132 - 120 = 12 (this remains)

Remainder is 12/15 = 4/5 = 0.8

